It's a Wonderful Lift

Get Your Axiomatizability Results for Weak or Timed Settings for Free

Wan Fokkink (VU University Amsterdam)

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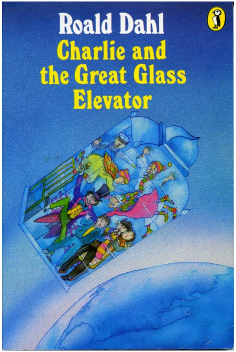

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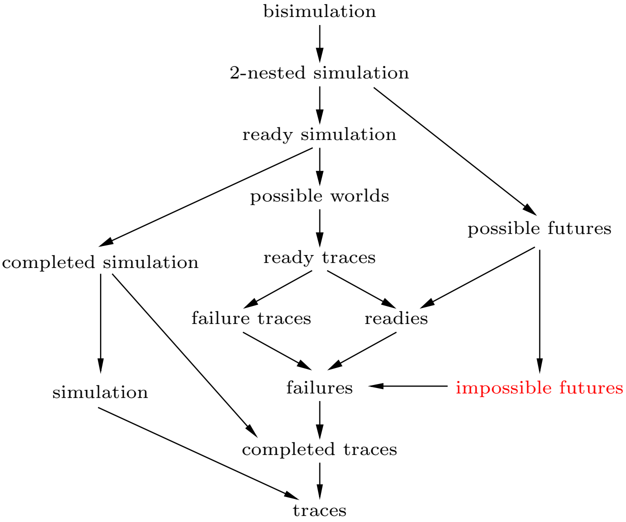
Rob's linear time - branching time spectrum I



BCCS(A) and BCCSP(A)

nil	$\mathbf{0}$	action prefix	at	$(a \in A)$	variables x
choice	$t + u$	silent step	τt	(only in BCCS)	

$$\frac{}{\alpha x \xrightarrow{\alpha} x} \qquad \frac{x \xrightarrow{\alpha} x'}{x + y \xrightarrow{\alpha} x'} \qquad \frac{y \xrightarrow{\alpha} y'}{x + y \xrightarrow{\alpha} y'}$$

$$\begin{array}{lrcl}
\text{A1} & x + y & = & y + x \\
\text{A2} & (x + y) + z & = & x + (y + z) \\
\text{A3} & x + x & = & x \\
\text{A4} & x + \mathbf{0} & = & x
\end{array}$$

An exhaustive exploration of axiomatizations

Finite ground-complete and ω-complete axiomatizations, and *impossibility* results, for the basic process algebra BCCSP, modulo the semantics in Rob's spectrum.

Taolue Chen, Wan Fokkink, Bas Luttik, Sumit Nain
On finite alphabets and infinite bases, *Information & Computation*, 2008

An axiomatization for a *preorder* can be transformed into one for the corresponding *equivalence*, preserving ground-/ω-completeness.

This works for semantics at least as coarse as ready simulation.

Luca Aceto, Wan Fokkink, Anna Ingolfsdottir
Ready to preorder: get your BCCSP axiomatization for free!, Proc. *CALCO'07*

Taolue Chen, Wan Fokkink, Rob van Glabbeek
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“How about impossible futures ?”

Taolue presented the work in the *I & C* paper at a PAM seminar.

Jos Baeten there asked: *“How about impossible futures ?”*

So (sigh) we started to study impossible futures semantics,
with some very surprising results.

The curious case of impossible futures

We gave a finite ground-complete axiomatization for BCCSP modulo impossible futures *preorder*.

This axiomatization is ω-complete if the alphabet is infinite.

We proved that for BCCSP modulo impossible futures *equivalence*, no finite, sound, ground-complete axiomatization exists.

Taloue Chen, Wan Fokkink
On the axiomatizability of impossible futures: preorder versus equivalence, *Proc. LICS'08*

This shows that the algorithm from the *CALCO'07* paper doesn't extend to impossible futures semantics.

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Weak impossible futures semantics

We lifted these results to weak impossible futures semantics.

Taloue Chen, Wan Fokkink, Rob van Glabbeek
On finite bases for weak semantics: Failures versus impossible futures, *Proc. SOFSEM'09*

The positive results were already obtained in

Marc Voorhoeve, Sjouke Mauw
Impossible futures and determinism, *Information Processing Letters*, 2001

A question by a reviewer

A reviewer wrote: "Proofs in the *LICS'08* and *SOFSEM'09* paper are similar, can (the proofs of) these results be related to each other ?"

The answer is *yes*, because the axioms

$$\begin{array}{lrcl} \mathrm{W1} & \alpha(\tau x + \tau y) & = & \alpha x + \alpha y \\ \mathrm{W2} & \tau x + y & = & \tau(x + y) + \tau x \end{array}$$

allow to eradicate all non-initial τ's.

This is ongoing research with Taolue and Rob.

Corresponding weak preorder

Given a strong preorder $\sqsubseteq_s$ over BCCSP(A).

A corresponding weak preorder $\sqsubseteq_w$ over BCCS(A) satisfies:

1. $\sqsubseteq_s$ and $\sqsubseteq_w$ coincide over τ-free *open* terms
2. renaming *initial* actions into τ is a precongruence for $\sqsubseteq_w$
3. W1-2 are sound for BCCS(A) modulo $\sqsubseteq_w$
4. if $t \sqsubseteq_w u$ where $t \stackrel{\tau}{\not\longrightarrow}$ and $u \stackrel{\tau}{\longrightarrow}$, then $x \sqsubseteq_w \tau x$
5. if $t \sqsubseteq_w u$ where $t \stackrel{\tau}{\longrightarrow}$ and $u \stackrel{\tau}{\not\longrightarrow}$, then $\tau x \sqsubseteq_w x$

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$\mathit{init}\text{-}\tau(t)$

$\mathit{init}\text{-}\tau(t)$ renames initial actions in open BCCSP term t into τ, and puts a τ in front of initial occurrences of variables.

$$\begin{aligned}
\mathit{init}\text{-}\tau(\mathbf{0}) &= \mathbf{0} \\
\mathit{init}\text{-}\tau(t+u) &= \mathit{init}\text{-}\tau(t) + \mathit{init}\text{-}\tau(u) \\
\mathit{init}\text{-}\tau(at) &= \tau t \\
\mathit{init}\text{-}\tau(x) &= \tau x
\end{aligned}$$

Generating an axiomatization for a weak semantics

Given an axiomatization E for BCCSP(A) modulo $\sqsubseteq_s$, with A1-4.

The axiomatization $\mathcal{A}(E)$ for BCCS(A) modulo a *corresponding* weak preorder $\sqsubseteq_w$ consists of:

- the axioms in E
- $\{init\text{-}\tau(t \preccurlyeq u) \mid t \preccurlyeq u \in E\}$
- W1-2
- if $t \sqsubseteq_w u$ where $t \xrightarrow{\tau}$ and $u \not\xrightarrow{\tau}$, then $\tau x \preccurlyeq x$ is included
- if $t \sqsubseteq_w u$ where $t \not\xrightarrow{\tau}$ and $u \xrightarrow{\tau}$, then $x \preccurlyeq \tau x$ is included

Generating an axiomatization for a weak semantics

Let $\sqsubseteq_s$ be a strong and $\sqsubseteq_w$ a *corresponding* weak preorder.

Let E be a sound and ground-complete axiomatization for BCCS(A) modulo $\sqsubseteq_s$.

Then $\mathcal{A}(E)$ is sound and ground-complete for BCCS(A) modulo $\sqsubseteq_w$.

If moreover E is ω-complete, then $\mathcal{A}(E)$ is ω-complete.

This approach can be tailored to *equivalences* (instead of preorders).

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Proof idea

Suppose $t \precsim_w u$.

Use W1-2 to transform t and u to t' and u' with only initial τ's.

Moreover, either $t' \not\xrightarrow{\tau}$ or $t' = \tau t_1 + \cdots + \tau t_k$ (likewise for u').

If $t' \xrightarrow{\tau}$ and $u' \xrightarrow{\tau}$:

- use W1 to rename the initial $\tau's$ into an $a \in A$.
- Use E to obtain a derivation in BCCSP(A).
- Transform this derivation to one between t' and u' by means of $\{init\text{-}\tau(t \preccurlyeq u) \mid t \preccurlyeq u \in E\}$.

If $t' \not\xrightarrow{\tau}$ and $u' \not\xrightarrow{\tau}$, simply use axioms in E.

If $t' \not\xrightarrow{\tau}$ and $u' \xrightarrow{\tau}$, use $x \preccurlyeq \tau x$.

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Corresponding weak preorders exist in the wild

The properties of corresponding preorders are satisfied by *weak*

- impossible futures $\precsim_{\mathrm{WIF}}$
- failures $\precsim_{\mathrm{WF}}$
- completed trace $\precsim_{\mathrm{WCT}}$
- trace $\precsim_{\mathrm{WT}}$

In particular, W1-2 and $x \preccurlyeq \tau x$ are sound for all four semantics.

$\tau x \precsim_{\mathrm{WT}} x$.

For the other three weak preorders, $t \sqsubseteq_w u$ and $t \xrightarrow{\tau}$ imply $u \xrightarrow{\tau}$.

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Application to weak failures / completed traces / traces

Finite, sound, ground-complete, ω-complete axiomatizations for BCCS modulo $\precsim_{\mathrm{WF}}$, $\precsim_{\mathrm{WCT}}$ and $\precsim_{\mathrm{WT}}$ were obtained from axiomatizations for the corresponding strong preorders.

By the aforementioned algorithm from

Luca Aceto, Wan Fokkink, Anna Ingolfsdottir
Ready to preorder: get your BCCSP axiomatization for free!, Proc. *CALCO'07*

these results extend to the associated *equivalences*.

Brave resistance

All known weak semantics at least as coarse as impossible futures are included.

All semantics ?

No, trace semantics for $|A| = 1$ offers brave resistance.

In that case $x \preccurlyeq ax$ is sound.

And the method doesn't work
if E contains an axiom
in which a variable occurs
both initial and non-initial.

Application to impossible futures preorder

There is a finite, sound, ground-complete axiomatization for BCCS(A) modulo $\precsim_{\mathrm{IF}}$.

This axiomatization is ω-complete if A is infinite.

Thus we obtain an axiomatization for BCCS(A) modulo $\precsim_{\mathrm{WIF}}$, which is ω-complete if A is infinite.

Application to impossible futures equivalence

No finite, sound, ground-complete axiomatization for BCCS(A) modulo $\sim_{\mathrm{WIF}}$ exists.

The infinite family of equations that underly this negative result are τ-free.

Hence the same negative result holds for BCCSP(A) modulo $\sim_{\mathrm{IF}}$.

Impossible futures preorder with a finite alphabet

If A is finite, then the equational theory of BCCS(A) modulo $\precsim_{\mathrm{WIF}}$ doesn't have a finite basis.

The infinite family of inequations that underly this negative result are τ-free.

Hence this same negative result holds for BCCSP(A) modulo $\precsim_{\mathrm{IF}}$.

Yet another question at a PAM seminar

In 2008, Mohammad Mousavi gave a PAM seminar on the need for a left merge to axiomatize parallel composition in a *timed* setting.

I conjectured this result could be obtained by lifting the classic result of Faron Moller from an untimed to a timed setting.

We developed a technique to lift *inaxiomatizability* results to an extended syntax, and applied it to timed and stochastic settings.

Luca Aceto, Wan Fokkink, Anna Ingolfsdottir, Mohammad Mousavi
Lifting non-finite axiomatizability results to extensions of process algebras, *Acta Informatica*, 2010

Lifting inaxiomatizability results

$\precsim_{\mathrm{o}}$ and $\precsim_{\mathrm{e}}$ are preorders over the original and extended syntax.

Let F be an infinite set of sound inequations over the original syntax which no finite sound axiomatization can prove.

Give a mapping $\mathcal{R}$ from the extended to the original syntax such that:

- if $t \precsim_{\mathrm{e}} u$, then $\mathcal{R}(t) \precsim_{\mathrm{o}} \mathcal{R}(u)$
- for all $v \preccurlyeq w \in F$ there are $t \precsim_{\mathrm{e}} u$ with $\mathcal{R}(t) = v$ and $\mathcal{R}(u) = w$
- if $E \vdash t \preccurlyeq u$, then $\mathcal{R}(E) \vdash \mathcal{R}(t) \preccurlyeq \mathcal{R}(u)$,
  for any axiomatization E over the extended syntax

There is an infinite set of sound inequations over the extended syntax which no finite sound axiomatization can prove.

Conclusion

Keep posing questions, both from the audience and as reviewers.

Open questions