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# ULTRAS

## A Uniform Framework for Process Models and Behavioral Equivalences

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Reporting on results of joint work with
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# Contents

# Introduction

Process Calculi (PCs), equipped with a formal operational semantics, are one of the most successful formalisms for modeling concurrent systems and proving their properties.

The operational semantics of PCs is defined by relying on Labeled Transition Systems (LTSs):

- the states of the transition systems are just PC terms
- the labels of the transitions connecting states represent the possible actions, or interactions, and their effects.

PCs often come equipped with observational mechanisms that permit equating (through behavioral equivalences) those systems that cannot be distinguished by external observations.

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# Introduction

Initially, PCs have been designed for modeling *qualitative* aspects of concurrent systems:

- to model functional (extensional) behavior
- to assess whether two systems have comparable behaviors

However, it was soon noticed that other aspects of concurrent systems, mainly related to systems performance, actions duration and probability, are at least as important as the functional ones.

Many variants of PCs have been introduced to take into account *quantitative* aspects of concurrent systems

- *deterministically timed* PCs;
- *probabilistic* PCs;
- *stochastically timed* PCs.

# Introduction

- The operational semantics of these (*timed*, *probabilistic* and *stochastic*) calculi has then been rendered in terms of richer LTSs especially with richer labels to model time, probabilities, rates
- new (*timed*, *probabilistic* and *stochastic*) behavioral relation have been introduced that have confront with combining nondeterminism and quantitative aspects.

With two of the organisers of this workshop, some times ago, we asked the following questions:

- Can we provide unifying definitions of the various models and equivalences?
- Do all instances of unifying definitions correspond to known ones or they give rise also to new models or equivalences?

# Our starting point

Our main sources of inspirations were two extensions of the LTS model, namely:

- Simple probabilistic automata [Segala, 1995], dealing with probabilistic systems.
- *Rate transition systems* [De Nicola-Latella-Loreti-Massink, 2009 and Klin-Sassone, 2008], dealing with stochastic systems.

Their commonality is in the transitions format: rather than mapping a state and a (possibly) decorated action into a next state they do

map a state and an action into a "distribution".

# Stochastic Process Calculi

A number of stochastic process calculi have been proposed in the last two decades. These are based on:

1. Labeled Transition Systems (LTS)
   - for providing compositional semantics of languages
   - for describing *qualitative properties*
2. Continuous Time Markov Chains (CTMC)
   - for analysing *quantitative properties*

Semantics of stochastic calculi have been provided by resorting to variants of the SOS approach but:

- there is no general framework for modelling the different formalisms
- it is rather difficult to appreciate differences and similarities of such semantics.

# Stochastic PCs - incomplete list

- TIPP (N. Glotz, U. Herzog, M. Rettelbach - 1993)
- Stochastic $\pi$-calculus (C. Priami - 1995, later with P. Quaglia)
- PEPA (J. Hillston - 1996)
- EMPA (M. Bernardo, R. Gorrieri - 1998)
- IMC (H. Hermanns - 2002)
- ...
- StoKlaim
- ...

More Calculi will come: Besides qualitative aspects of distributed systems it more and more important that performance and dependability be addressed to deal, e.g., with issues related to quality of service.
Meta Models are essential: They will avoid developing ad hoc theories for each new calculus

# Stochastic Semantics

## Associating CTMC to Stochastic PC

- To model the stochastic behaviour of processes a CTMC is associated to each process term;
- To get a CTMC from a term, one needs to compute synchronization rates while taking into account transition multiplicity, for determining correct execution rate

## The race condition problem

- Process Calculi:
  $$\alpha.P + \alpha.P \quad = \quad \alpha.P$$
- Stochastic Process Calculi:
  $$\alpha^{\lambda}.P + \alpha^{\lambda}.P \quad = \quad \alpha^{2\lambda}.P$$

# RTS: A unifying approach

## Ad hoc solutions for duplicated races

Almost each calculus has an ad hoc implementation of the race condition principle of CTMC:

- multi-relations *[e.g. PEPA, IML]*
- proved transition systems *[e.g. TIPP, S$\pi$C]*
- LTS with numbered transitions *[e.g. LCTMC]*
- unique rate names *[e.g. StoKLAIM]*

## Definition (Rate Transition Systems)

A rate transition system is a triple $(S, A, \longrightarrow)$ where:

- $S$ is a set of states;
- $A$ is a set of transition labels;
- $\rightarrow \subseteq S \times A \times [S \rightarrow \mathbb{R}_{\geq 0}]$

# Semantics of stochastic process calculi

Stochastic semantics of process calculi is defined by means of a transition relation $\longrightarrow$ that associates to a pair $(P, \alpha)$ - consisting of process and an action - a total function $(\mathscr{P}, \mathscr{Q}, \ldots)$ that assigns a non-negative real number to each process of the calculus. Value 0 is assigned to unreachable processes.

$P \xrightarrow{\alpha} \mathscr{P}$ means that, for a generic process $Q$:

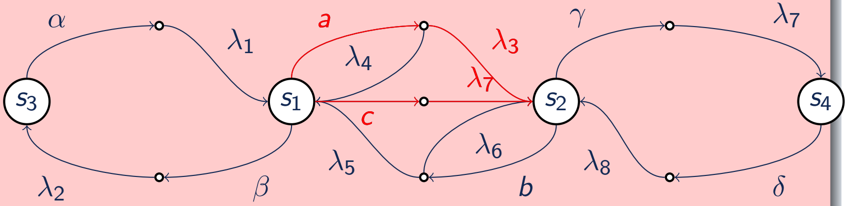
- if $\mathscr{P}(Q) = x\ (\neq 0)$ then $Q$ is reachable from $P$ via the execution of $\alpha$ with rate/(weight) $x$
- if $\mathscr{P}(Q) = 0$ then $Q$ is not reachable from $P$ via $\alpha$

We have that if $P \xrightarrow{\alpha} \mathscr{P}$ then

- $\oplus\mathscr{P} = \sum_Q \mathscr{P}(Q)$ represents the total rate/weight of $\alpha$ in $P$.

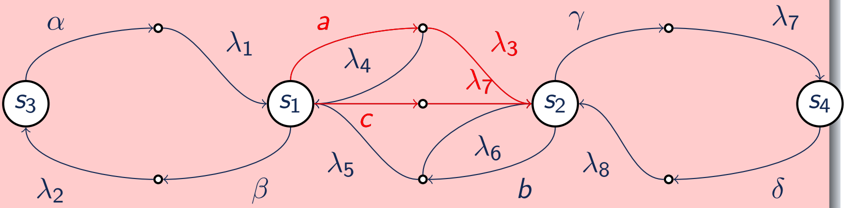
# From RTS to CTMC

## An RTS:

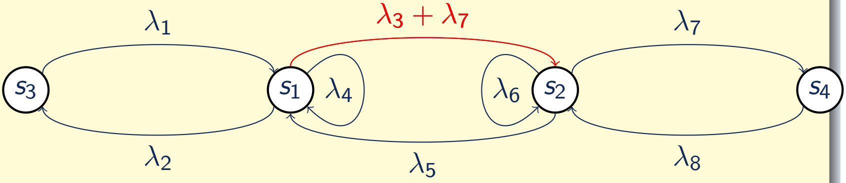


# From RTS to CTMC

## An RTS:




## The corresponding CTMC:



# The ULTRAS Model

- $(D, \sqsubseteq_D, \bot_D)$: preordered set equipped with a minimum denoted by $\bot_D$, with each value representing a degree of one-step reachability.
- A Uniform Labeled Transition System on $(D, \sqsubseteq_D, \bot_D)$, or $D$-ULTRAS, is a triple $\mathcal{U} = (S, A, \longrightarrow)$ where:
  - $S$ is an at most countable set of states.
  - $A$ is a countable set of transition-labeling actions.
  - $\longrightarrow \subseteq S \times A \times [S \to D]$ is a transition relation.
- $\mathcal{U}$ is *functional* iff $\longrightarrow$ is a function from $S \times A$ to $[S \to D]$.
- Given a transition $s \xrightarrow{a} \Delta$, function $\Delta$ represents the distribution of reachability of all possible states from $s$ via that transition.
- If $\Delta(s') = \bot_D$, then $s'$ is not reachable from $s$ via that transition.

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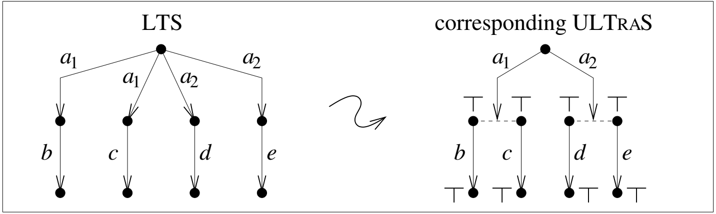
# Nondeterministic Models as ULTraS

- An LTS can be encoded as a functional $\mathbb{B}$-ULTraS, where $\mathbb{B} = \{\bot, \top\}$ is the support set of the Boolean algebra with $\bot \sqsubseteq_{\mathbb{B}} \top$.
- An LTS is a triple $(S, A, \longrightarrow)$ where:
  - $S$ is an at most countable set of states.
  - $A$ is a countable set of transition-labeling actions.
  - $\longrightarrow \subseteq S \times A \times S$ is a transition relation.
- Corresponding functional $\mathbb{B}$-ULTraS $\mathcal{U} = (S, A, \longrightarrow_{\mathcal{U}})$:
  - $s \xrightarrow{a}_{\mathcal{U}} \Delta_{s,a}$ for all $s \in S$ and $a \in A$.
  - $\Delta_{s,a}(s') = \begin{cases} \top & \text{if } s \xrightarrow{a} s' \\ \bot & \text{if } (s, a, s') \notin \longrightarrow \end{cases}$ for all $s' \in S$.

# From LTS to ULTraS

- External and internal nondeterminism are encoded differently:



# Probabilistic Models as ULTRAS

Different models featuring probabilities and different levels of nondeterminism.

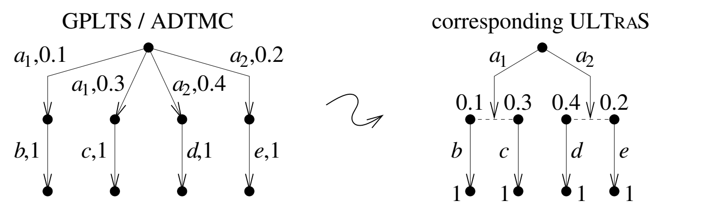
- A Generative Probabilistic System (GPLTS) or action-labeled discrete-time Markov chain – ADTMC can be encoded as a functional $\mathbb{R}_{[0,1]}$-ULTRAS with the usual $\leq$.
- A Reactive Probabilistic System (RPLTS) or discrete-time Markov decision process – MDP can be encoded as a functional $\mathbb{R}_{[0,1]}$-ULTRAS with the usual $\leq$.
- A Nondeterministic and Probabilistic System (NPLTS) which is an MDP with internal nondeterminism can be encoded as an $\mathbb{R}_{[0,1]}$-ULTRAS with the usual $\leq$.

# GPLTS as ULTRAS

- A generative probabilistic LTS is a triple $(S, A, \longrightarrow)$ where:
  - $S$ is an at most countable set of states.
  - $A$ is a countable set of transition-labeling actions.
  - $\longrightarrow \subseteq S \times A \times \mathbb{R}_{(0,1]} \times S$ is a transition relation.
  - Whenever $s \xrightarrow{a,p_1} s'$ and $s \xrightarrow{a,p_2} s'$, then $p_1 = p_2$.
  - $\sum \{\!| \, p \in \mathbb{R}_{(0,1]} \mid \exists a \in A. \exists s' \in S. \, s \xrightarrow{a,p} s' \, |\!\} \in \{0, 1\}$ for all $s \in S$.
- Corresponding functional $\mathbb{R}_{[0,1]}$-ULTRAS $\mathcal{U} = (S, A, \longrightarrow_{\mathcal{U}})$:
  - $s \xrightarrow{a}_{\mathcal{U}} \Delta_{s,a}$ for all $s \in S$ and $a \in A$.
  - $\Delta_{s,a}(s') = \begin{cases} p & \text{if } s \xrightarrow{a,p} s' \\ 0 & \text{if } \nexists p \in \mathbb{R}_{(0,1]}. \, s \xrightarrow{a,p} s' \end{cases}$ for all $s' \in S$.

# From GPLTS to ULTRAS

- External and internal probabilistic choices, probability *sub*distributions:

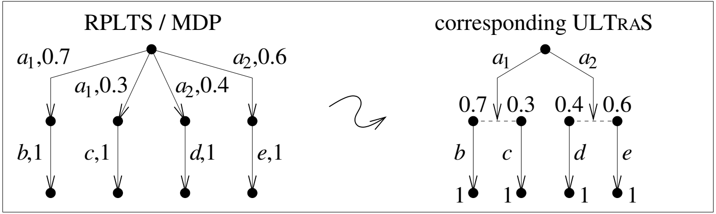


# RPLTS as ULTRAS

- A reactive probabilistic LTS is a triple $(S, A, \longrightarrow)$ where:
  - $S$ is an at most countable set of states.
  - $A$ is a countable set of transition-labeling actions.
  - $\longrightarrow \subseteq S \times A \times \mathbb{R}_{(0,1]} \times S$ is a transition relation.
  - Whenever $s \xrightarrow{a,p_1} s'$ and $s \xrightarrow{a,p_2} s'$, then $p_1 = p_2$.
  - $\sum \{\!| p \in \mathbb{R}_{(0,1]} \mid \exists s' \in S.\, s \xrightarrow{a,p} s' |\!\} \in \{0,1\}$ for all $s \in S$ and $a \in A$.
- Corresponding functional $\mathbb{R}_{[0,1]}$-ULTRAS $\mathcal{U} = (S, A, \longrightarrow_{\mathcal{U}})$:
  - $s \xrightarrow{a}_{\mathcal{U}} \Delta_{s,a}$ for all $s \in S$ and $a \in A$.
  - $\Delta_{s,a}(s') = \begin{cases} p & \text{if } s \xrightarrow{a,p} s' \\ 0 & \text{if } \nexists p \in \mathbb{R}_{(0,1]}.\, s \xrightarrow{a,p} s' \end{cases}$ for all $s' \in S$.

# From RPLTS to ULTRAS

- External nondeterminism & internal probabilistic choices:

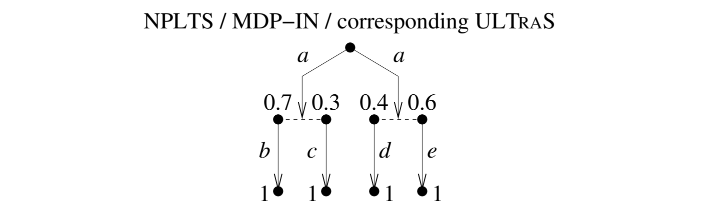


# NPLTS: Nondeterministic and Probabilistic Systems

- A nondeterministic and probabilistic LTS is a triple $(S, A, \longrightarrow)$ where:
  - $S$ is an at most countable set of states.
  - $A$ is a countable set of transition-labeling actions.
  - $\longrightarrow \subseteq S \times A \times [S \to \mathbb{R}_{[0,1]}]$ is a transition relation.
  - $\sum_{s' \in S} \Delta(s') = 1$ for all $s \xrightarrow{a} \Delta$.
- The corresponding $\mathbb{R}_{[0,1]}$-ULTRAS is $(S, A, \longrightarrow)$ itself.
- Not functional due to the coexistence of internal nondeterminism and probabilistic choices.

# NPLTS

- External/internal nondeterminism & internal probabilistic choices:



# Stochastic Models as ULTRAS

- Models featuring rates and different levels of nondeterminism.
- Rates encompass both probabilistic and timing aspects.
- A GMLTS (or action-labeled continuous-time Markov chain – ACTMC) can be encoded as a functional $\mathbb{R}_{\geq 0}$-ULTRAS with the usual $\leq$.
- An RMLTS (or continuous-time Markov decision process – CTMDP) can be encoded as a functional $\mathbb{R}_{\geq 0}$-ULTRAS with the usual $\leq$.
- An NMLTS (or CTMDP with internal nondeterminism) can be encoded as an $\mathbb{R}_{\geq 0}$-ULTRAS with the usual $\leq$.

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# Three semantics for CSP

## Basic CSP

$$P \ ::= \ a.P \mid P_1 + P_2 \mid P_1 \parallel_L P_2 \mid B$$

$P_1 + P_2$ models a process that may behave either as $P_1$ or as $P_2$, while $P_1 \parallel_L P_2$ models the parallel execution of $P_1$ and $P_2$, which synchronize on actions in $L$.

## Probabilistic CSP

$$P \ ::= \ a.P \mid P_1 +_p P_2 \mid P_1 \parallel_L{}^p P_2 \mid B$$

$P_1 +_p P_2$ models a process that behaves as $P_1$ with probability $p$ or as $P_2$ with probability $1 - p$. In $P_1 \parallel_L{}^p P_2$ the value $p$ regulates the interleaving of $P_1$ and $P_2$.

## Stochastic CSP

$$P \ ::= \ (a, \lambda).P \mid P + P \mid P \parallel_L P \mid B$$

Component $(a, \lambda).P$ models a process that can perform action $a$ at rate $\lambda$ and then behaves like $P$.

# Interleaving semantics for CSP

$$\frac{}{a.P \xrightarrow{a} [P \mapsto \top]}\ \textsc{Act} \qquad \frac{b \neq a}{a.P \xrightarrow{b} [\,]}\ \emptyset\text{-}\textsc{Act} \qquad \frac{B \triangleq P \quad P \xrightarrow{a} \mathscr{D}}{B \xrightarrow{a} \mathscr{D}}\ \textsc{Call}$$

$$\frac{P_1 \xrightarrow{a} \mathscr{D}_1 \quad P_2 \xrightarrow{a} \mathscr{D}_2}{P_1 + P_2 \xrightarrow{a} \mathscr{D}_1 \vee \mathscr{D}_2}\ \textsc{Sum}$$

$$\frac{P_1 \xrightarrow{a} \mathscr{D}_1 \quad P_2 \xrightarrow{a} \mathscr{D}_2 \quad a \in L}{P_1 \parallel_L P_2 \xrightarrow{a} \mathscr{D}_1 \parallel_L \mathscr{D}_2}\ \textsc{Coop}$$

$$\frac{P_1 \xrightarrow{a} \mathscr{D}_1 \quad P_2 \xrightarrow{a} \mathscr{D}_2 \quad a \notin L}{P_1 \parallel_L P_2 \xrightarrow{a} (\mathscr{D}_1 \parallel_L P_2) \vee (P_1 \parallel_L \mathscr{D}_2)}\ \textsc{Int}$$

Table: CSP operational semantic rules

# Probabilistic Semantics for CSP

$$\frac{}{a.P \xrightarrow{a} [P \mapsto 1]}\ \textsc{Act} \qquad \frac{b \neq a}{a.P \xrightarrow{b} [\,]}\ \emptyset\text{-}\textsc{Act} \qquad \frac{B \triangleq P \quad P \xrightarrow{a} \mathscr{D}}{B \xrightarrow{a} \mathscr{D}}\ \textsc{Call}$$

$$\frac{P_1 \xrightarrow{a} \mathscr{D}_1 \quad P_2 \xrightarrow{a} \mathscr{D}_2}{P_1 +_p P_2 \xrightarrow{a} \frac{p\cdot\oplus\mathscr{D}_1}{p\cdot\oplus\mathscr{D}_1+(1-p)\cdot\oplus\mathscr{D}_2}\cdot\mathscr{D}_1 + \frac{(1-p)\cdot\oplus\mathscr{D}_2}{p\cdot\oplus\mathscr{D}_1+(1-p)\cdot\oplus\mathscr{D}_2}\cdot\mathscr{D}_2}\ \textsc{Sum}$$

$$\frac{P_1 \xrightarrow{a} \mathscr{D}_1 \quad P_2 \xrightarrow{a} \mathscr{D}_2 \quad a \in L}{P_1 \,\|_L{}^p\, P_2 \xrightarrow{a} \mathscr{D}_1 \,\|_L\, \mathscr{D}_2}\ \textsc{Coop}$$

$$\frac{P_1 \xrightarrow{a} \mathscr{D}_1 \quad P_2 \xrightarrow{a} \mathscr{D}_2 \quad a \notin L}{P_1 \,\|_L{}^p\, P_2 \xrightarrow{a} \frac{p\cdot\oplus\mathscr{D}_1}{p\cdot\oplus\mathscr{D}_1+(1-p)\cdot\oplus\mathscr{D}_2}\cdot(\mathscr{D}_1 \,\|_L\, P_2) + \frac{(1-p)\cdot\oplus\mathscr{D}_2}{p\oplus\mathscr{D}_1+(1-p)\cdot\oplus\mathscr{D}_2}\cdot(P_1 \,\|_L\, \mathscr{D}_2)}\ \textsc{Int}$$

Table: PCSP operational semantic rules

# Stochastic Semantics for CSP

$$\frac{}{(a,\lambda).P \xrightarrow{a} [P \mapsto \lambda]}\ \textsc{Act} \qquad \frac{a \neq b}{(a,\lambda).P \xrightarrow{b} [\,]}\ \emptyset\text{-}\textsc{Act} \qquad \frac{B \xrightarrow{a} \mathscr{D} \quad B \triangleq P}{P \xrightarrow{a} \mathscr{D}}\ \textsc{Call}$$

$$\frac{P_1 \xrightarrow{a} \mathscr{D}_1 \quad P_2 \xrightarrow{a} \mathscr{D}_2}{P_1 + P_2 \xrightarrow{a} \mathscr{D}_1 + \mathscr{D}_2}\ \textsc{Sum}$$

$$\frac{P_1 \xrightarrow{a} \mathscr{D}_1 \quad P_2 \xrightarrow{a} \mathscr{D}_2 \quad a \in L}{P_1 \parallel_L P_2 \xrightarrow{a} \frac{\min\{\oplus\mathscr{D}_1, \oplus\mathscr{D}_2\}}{\oplus\mathscr{D}_1 \cdot \oplus\mathscr{D}_2} \cdot (\mathscr{D}_1 \parallel_L \mathscr{D}_2)}\ \textsc{Coop}$$

$$\frac{P_1 \xrightarrow{a} \mathscr{D}_1 \quad P_2 \xrightarrow{a} \mathscr{D}_2 \quad a \notin L}{P_1 \parallel_L P_2 \xrightarrow{a} (\mathscr{D}_1 \parallel_L P_2) + (P_1 \parallel_L \mathscr{D}_2)}\ \textsc{Int}$$

Table: PEPA operational semantic rules

# Behavioral Equivalences for ULTRAS

- $(M, \sqsubseteq_M, \bot_M)$: preordered set equipped with a minimum denoted by $\bot_M$, with each value representing a degree of multi-step reachability.
- A $M$-measure function on $(M, \sqsubseteq_M, \bot_M)$ for $\mathcal{U} = (S, A, \longrightarrow)$, is a function $\mathcal{M}_M : S \times A^* \times 2^S \to M$ such that the value of $\mathcal{M}_M(s, \alpha, S')$ is defined by induction on $|\alpha|$ and depends only on the reachability of a state in $S'$ from state $s$ through computations labeled with trace $\alpha$.
- A measure function assumes existence of two operators:
  - A *multiplicative operator* $\otimes$ that combines into an $M$-value the $D$-values corresponding to each individual step along a single computation labeled with trace $\alpha$ that goes from $s$ to $S'$.
  - An *additive operator* $\oplus$ that combines the $M$-values computed for each considered computation with the previous operator.

- $D$ and $M$ are not necessarily the same set.
- A $D$-value $\Delta(s')$ is related to one-step reachability.
- An $M$-value $\mathcal{M}_M(s, \alpha, S')$ is related to multi-step reachability.
- Testing equivalence for LTS models: the $M$-value will be a pair of $\mathbb{B}$-values – *rather than a single* $\mathbb{B}$*-value* – to take into account the possibility and the necessity of reaching $S'$ from $s$ after $\alpha$.
- Equivalences for NPLTS models: the $M$-value will be a nonemtpy set of $\mathbb{R}_{[0,1]}$-values – *rather than a single* $\mathbb{R}_{[0,1]}$*-value* – to take into account all possible ways of resolving internal nondeterminism.
- Equivalences for stochastic models: the $M$-value will be an $\mathbb{R}_{[0,1]}$-valued function – *rather than a single* $\mathbb{R}_{\geq 0}$*-value* – representing for each possible end-to-end/step-by-step deadline the probability (or set of probabilities) of reaching $S'$ from $s$ via $\alpha$ within the considered deadline.

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# Trace Equivalence

**Definition**

We say that $s_1, s_2 \in S$ are $\mathcal{M}_M$-trace equivalent, written $s_1 \sim_{\mathrm{Tr},\mathcal{M}_M} s_2$, iff for all traces $\alpha \in A^*$:

$$\mathcal{M}_M(s_1, \alpha, S) = \mathcal{M}_M(s_2, \alpha, S)$$

**Remark**

Using the entire $S$ as set of destination states means that destination states are not important; what matters is the capability of executing $\alpha$.

# Bisimilarity

## Definition

- An equivalence relation $\mathcal{B}$ over $S$ is an $\mathcal{M}_M$-bisimulation iff, whenever $(s_1, s_2) \in \mathcal{B}$, then for all actions $a \in A$ and groups of equivalence classes $\mathcal{G} \in 2^{S/\mathcal{B}}$:

$$\mathcal{M}_M(s_1, a, \bigcup \mathcal{G}) = \mathcal{M}_M(s_2, a, \bigcup \mathcal{G})$$

- We say that $s_1, s_2 \in S$ are $\mathcal{M}_M$-bisimilar, written $s_1 \sim_{\mathrm{B},\mathcal{M}_M} s_2$, iff there exists an $\mathcal{M}_M$-bisimulation $\mathcal{B}$ over $S$ such that $(s_1, s_2) \in \mathcal{B}$.

## Remark

The definition of bisimulation equivalence concentrates on traces of length 1 and does take into account destination states.

# Testing Equivalence

- A *D*-observation system is a $D$-ULTRAS $\mathcal{O} = (O, A, \longrightarrow_{\mathcal{O}})$ where $O$ contains a distinguished success state denoted by $\omega$ such that, whenever $\omega \stackrel{a}{\longrightarrow} \Delta$, then $\Delta(o) = \perp_D$ for all $o \in O$.
- Need $D$-valued function $\delta$ for the interaction system $\mathcal{I}(\mathcal{U}, \mathcal{O})$ to combine the target distributions of the synchronizing transitions of $\mathcal{U}$ and $\mathcal{O}$, which preserves $\perp_D$ and is injective.
- States are configurations $(s, o)$ that are successful when $o = \omega$: $\mathcal{S}^{\delta}(\mathcal{U}, \mathcal{O})$ denotes all succesfull configuration.
- $s_1, s_2 \in S$ are $\mathcal{M}_M^{\delta}$-testing equivalent, written $s_1 \sim_{\mathrm{Te}, \mathcal{M}_M^{\delta}} s_2$, iff for every $D$-observation system $\mathcal{O} = (O, A, \longrightarrow_{\mathcal{O}})$ with initial state $o \in O$ and for all $\alpha \in A^*$:

$$\mathcal{M}_M^{\delta,\mathcal{O}}((s_1, o), \alpha, \mathcal{S}^{\delta}(\mathcal{U}, \mathcal{O})) = \mathcal{M}_M^{\delta,\mathcal{O}}((s_2, o), \alpha, \mathcal{S}^{\delta}(\mathcal{U}, \mathcal{O}))$$

# Retrieving Existing Behavioral Equivalences

- Most of the bisimulation, trace, and testing equivalences appeared in the literature since the 80's are captured by our general framework except when internal nondeterminism coexist with probabilities or rates.
- For NPLTS models, we have obtained equivalences different from those appeared in the literature, which possess interesting properties:
  - Bisimilarity is characterised by PML (unlike the one in [Segala-Lynch, 1994])
  - Trace equivalence is compositional (unlike the one in [Segala, 1995])
  - Testing equivalence is fully compatible with the original nondeterministic one (unlike the one in [Yi-Larsen, 1992; Jonsson-Yi, 1995; Segala, 1996; Deng-VanGlabbeek-Hennessy-Morgan, 2008].)
- For NMLTS models, there are no equivalences defined in the literature, hence we have provided them for the first time.

# Nondeterministic Systems

- Nondeterministic behavioral equivalences:

| | | | |
|---|---|---|---|
| LTS | $\sim_{\mathrm{B}}$ [Park, 1981][Milner, 1984]<br>$\sim_{\mathrm{Tr}}$ [Brookes-Hoare-Roscoe, 1984]<br>$\sim_{\mathrm{Te}}$ [De Nicola-Hennessy, 1984] | $\sim_{\mathrm{B},\mathcal{M}_{\mathbb{B},\vee}}$<br>$\sim_{\mathrm{Tr},\mathcal{M}_{\mathbb{B},\vee}}$<br>$\sim_{\mathrm{Te},\mathcal{M}^{\mathrm{LC}}_{\mathbb{B}\times\mathbb{B}}}$ | functional $\mathbb{B}$-ULTraS |

- Nondeterministic measure functions:

$$\mathcal{M}_{\mathbb{B},\vee}(s,\alpha,S') = \begin{cases} \bigvee_{s'\in S \text{ s.t. } \Delta_{s,a}(s')\neq\bot} \mathcal{M}_{\mathbb{B},\vee}(s',\alpha',S') & \alpha = a\circ\alpha' \\ \top;\bot & \alpha=\varepsilon,\ s\in S'? \end{cases}$$

$$\mathcal{M}_{\mathbb{B},\wedge}(s,\alpha,S') = \begin{cases} \bigwedge_{s'\in S \text{ s.t. } \Delta_{s,a}(s')\neq\bot} \mathcal{M}_{\mathbb{B},\wedge}(s',\alpha',S') & \alpha = a\circ\alpha' \\ \top;\bot & \alpha=\varepsilon,\ s\in S'? \end{cases}$$

$$\mathcal{M}_{\mathbb{B}\times\mathbb{B}}(s,\alpha,S') = (\mathcal{M}_{\mathbb{B},\vee}(s,\alpha,S'),\mathcal{M}_{\mathbb{B},\wedge}(s,\alpha,S'))$$

# Probabilistic Systems

- Probabilistic behavioral equivalences:

| | | | |
|---|---|---|---|
| GPLTS | $\sim_{\mathrm{PB}}$<br>$\sim_{\mathrm{PTr}}$<br>$\sim_{\mathrm{PTe}}$ | $\sim_{\mathrm{B},\mathcal{M}_{\mathbb{R}_{[0,1]}}}$<br>$\sim_{\mathrm{Tr},\mathcal{M}_{\mathbb{R}_{[0,1]}}}$<br>$\sim_{\mathrm{Te},\mathcal{M}^{\mathrm{NPM}}_{\mathbb{R}_{[0,1]}}}$ | functional $\mathbb{R}_{[0,1]}$-ULTRAS<br>such that for all $s \in S$<br>$\sum_{a \in A} \sum_{s' \in S} \Delta_{s,a}(s') \in \{0,1\}$ |
| RPLTS | $\sim_{\mathrm{PB}}$ [Larsen-Skou, 1991]<br>$\sim_{\mathrm{PTr}}$<br>$\sim_{\mathrm{PTe}}$ | $\sim_{\mathrm{B},\mathcal{M}_{\mathbb{R}_{[0,1]}}}$<br>$\sim_{\mathrm{Tr},\mathcal{M}_{\mathbb{R}_{[0,1]}}}$<br>$\sim_{\mathrm{Te},\mathcal{M}^{\mathrm{PM}}_{\mathbb{R}_{[0,1]}}}$ | functional $\mathbb{R}_{[0,1]}$-ULTRAS<br>such that for all $s \in S$ and $a \in A$<br>$\sum_{s' \in S} \Delta_{s,a}(s') \in \{0,1\}$ |
| NPLTS | $\sim_{\mathrm{PB,N}}$<br>$\sim_{\mathrm{PTr,N}}$<br>$\sim_{\mathrm{PTe,N}}$ | $\sim_{\mathrm{B},\mathcal{M}_{2^{\mathbb{R}_{[0,1]}}}}$<br>$\sim_{\mathrm{Tr},\mathcal{M}_{2^{\mathbb{R}_{[0,1]}}}}$<br>$\sim_{\mathrm{Te},\mathcal{M}^{\mathrm{PM}}_{2^{\mathbb{R}_{[0,1]}}}}$ | $\mathbb{R}_{[0,1]}$-ULTRAS<br>such that for all $s \xrightarrow{a} \Delta$<br>$\sum_{s' \in S} \Delta(s') = 1$ |

- Probabilistic measure functions:

$$\mathcal{M}_{\mathbb{R}_{[0,1]}}(s, \alpha, S') = \begin{cases} \sum_{s' \in S} \Delta_{s,a}(s') \cdot \mathcal{M}_{\mathbb{R}_{[0,1]}}(s', \alpha', S') & \alpha = a \circ \alpha' \\ 1; 0 & \alpha = \varepsilon,\ s \in S'? \end{cases}$$

$$\mathcal{M}_{2^{\mathbb{R}_{[0,1]}}}(s, \alpha, S') = \begin{cases} \bigcup_{s \xrightarrow{a} \Delta} \{ \sum_{s' \in S} \Delta(s') \cdot p_{s'} \mid p_{s'} \in \mathcal{M}_{2^{\mathbb{R}_{[0,1]}}}(s', \alpha', S')\} & \alpha = a \circ \alpha' \\ \{1\}; \{0\} & \alpha = \varepsilon,\ s \in S'? \end{cases}$$

# Stochastic Systems

- Stochastic behavioral equivalences:

| | | | |
|---|---|---|---|
| GMLTS | $\sim_{\mathrm{MB}}$ [Hillston, 1996] <br> $\sim_{\mathrm{MTr,ete}}$ $\sim_{\mathrm{MTr,sbs}}$ <br> $\sim_{\mathrm{MTe,ete}}$ $\sim_{\mathrm{MTe,sbs}}$ | $\sim_{\mathrm{B},\mathcal{M}_{\mathrm{ete}}}$ $\sim_{\mathrm{B},\mathcal{M}_{\mathrm{sbs}}}$ <br> $\sim_{\mathrm{Tr},\mathcal{M}_{\mathrm{ete}}}$ $\sim_{\mathrm{Tr},\mathcal{M}_{\mathrm{sbs}}}$ <br> $\sim_{\mathrm{Te},\mathcal{M}^{\mathrm{RM}}_{\mathrm{ete}}}$ $\sim_{\mathrm{Te},\mathcal{M}^{\mathrm{RM}}_{\mathrm{sbs}}}$ | functional $\mathbb{R}_{\geq 0}$-ULTRAS |
| RMLTS | $\sim_{\mathrm{MB}}$ <br> $\sim_{\mathrm{MTr,ete,R}}$ $\sim_{\mathrm{MTr,sbs,R}}$ <br> $\sim_{\mathrm{MTe,ete,R}}$ $\sim_{\mathrm{MTe,sbs,R}}$ | $\sim_{\mathrm{B},\mathcal{M}_{\mathrm{ete,R}}}$ $\sim_{\mathrm{B},\mathcal{M}_{\mathrm{sbs,R}}}$ <br> $\sim_{\mathrm{Tr},\mathcal{M}_{\mathrm{ete,R}}}$ $\sim_{\mathrm{Tr},\mathcal{M}_{\mathrm{sbs,R}}}$ <br> $\sim_{\mathrm{Te},\mathcal{M}^{\mathrm{RM}}_{\mathrm{ete,R}}}$ $\sim_{\mathrm{Te},\mathcal{M}^{\mathrm{RM}}_{\mathrm{sbs,R}}}$ | functional $\mathbb{R}_{\geq 0}$-ULTRAS |
| NMLTS | $\sim_{\mathrm{MB,N}}$ <br> $\sim_{\mathrm{MTr,ete,N}}$ $\sim_{\mathrm{MTr,sbs,N}}$ <br> $\sim_{\mathrm{MTe,ete,N}}$ $\sim_{\mathrm{MTe,sbs,N}}$ | $\sim_{\mathrm{B},\mathcal{M}_{\mathrm{ete,N}}}$ $\sim_{\mathrm{B},\mathcal{M}_{\mathrm{sbs,N}}}$ <br> $\sim_{\mathrm{Tr},\mathcal{M}_{\mathrm{ete,N}}}$ $\sim_{\mathrm{Tr},\mathcal{M}_{\mathrm{sbs,N}}}$ <br> $\sim_{\mathrm{Te},\mathcal{M}^{\mathrm{RM}}_{\mathrm{ete,N}}}$ $\sim_{\mathrm{Te},\mathcal{M}^{\mathrm{RM}}_{\mathrm{sbs,N}}}$ | $\mathbb{R}_{\geq 0}$-ULTRAS such that <br> for all $s \xrightarrow{a} \Delta$ <br> $\sum_{s' \in S} \Delta(s') > 0$ |

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# Stochastic Systems ctd.

- Stochastic measure functions:

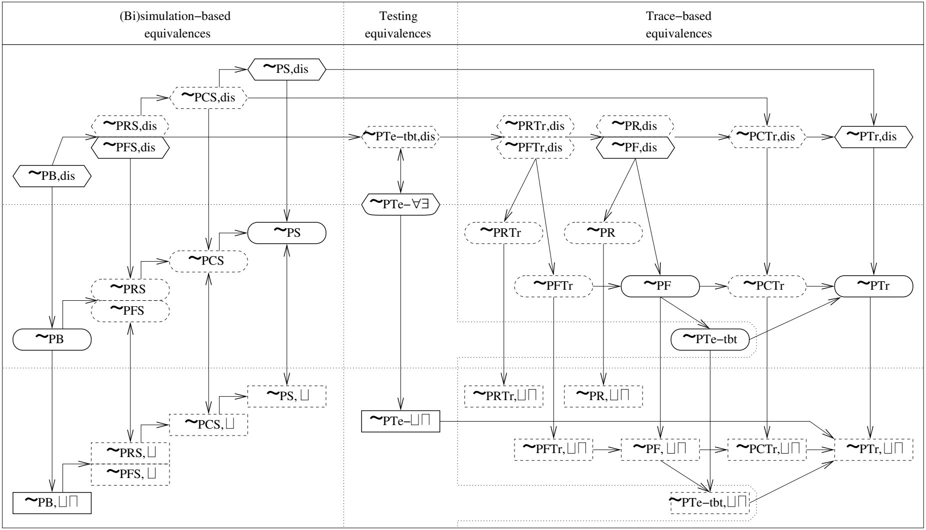
$$\mathcal{M}_{\mathrm{ete,R}}(s,\alpha,S')(t) = \begin{cases} \int_0^t \mathrm{E}_a(s)\cdot \mathrm{e}^{-\mathrm{E}_a(s)\cdot x}\cdot \sum_{s'\in S} \frac{\Delta_{s,a}(s')}{\mathrm{E}_a(s)}\cdot \mathcal{M}_{\mathrm{ete,R}}(s',\alpha',S')(t-x)\,\mathrm{d}x & \alpha = a\circ\alpha',\ \mathrm{E}_a(s)>0 \\ 1;0 & \alpha=\varepsilon,\ s\in S'? \end{cases}$$

$$\mathcal{M}_{\mathrm{sbs,R}}(s,\alpha,S')(\theta) = \begin{cases} (1-\mathrm{e}^{-\mathrm{E}_a(s)\cdot t})\cdot \sum_{s'\in S} \frac{\Delta_{s,a}(s')}{\mathrm{E}_a(s)}\cdot \mathcal{M}_{\mathrm{sbs,R}}(s',\alpha',S')(\theta') & \alpha = a\circ\alpha',\ \theta=t\circ\theta',\ \mathrm{E}_a(s)>0 \\ 1;0 & \alpha=\varepsilon,\ s\in S'? \end{cases}$$

$$\mathcal{M}_{\mathrm{ete,N}}(s,\alpha,S')(t) = \begin{cases} \bigcup_{s \xrightarrow{a} \Delta} \{\int_0^t \Delta(S)\cdot \mathrm{e}^{-\Delta(S)\cdot x}\cdot \sum_{s'\in S} \frac{\Delta(s')}{\Delta(S)}\cdot p_{s'}\,\mathrm{d}x \mid p_{s'}\in \mathcal{M}_{\mathrm{ete,N}}(s',\alpha',S')(t-x)\} & \alpha = a\circ\alpha' \\ \{1\};\{0\} & \alpha=\varepsilon,\ s\in S'? \end{cases}$$

$$\mathcal{M}_{\mathrm{sbs,N}}(s,\alpha,S')(\theta) = \begin{cases} \bigcup_{s \xrightarrow{a} \Delta} \{(1-\mathrm{e}^{-\Delta(S)\cdot t})\cdot \sum_{s'\in S} \frac{\Delta(s')}{\Delta(S)}\cdot p_{s'} \mid p_{s'}\in \mathcal{M}_{\mathrm{sbs,N}}(s',\alpha',S')(\theta')\} & \alpha = a\circ\alpha',\ \theta=t\circ\theta' \\ \{1\};\{0\} & \alpha=\varepsilon,\ s\in S'? \end{cases}$$

# A spectrum with deterministic schedulers



# Future Work

- Determining the unifying equivalence that yield classical bisimilation-based equivalence (Segala-Lynch) also for NPLTS
- Defining an ULTRAS-based operational semantics of richer process calculi for investigating their relative expressiveness.
- Studying a generic process algebra together with uniform results for congruence properties & equational/logical characterizations, as well as uniform algorithms for equivalence checking and model checking.
- Providing uniform definitions of weak behavioral equivalences.
- Extending ULTRAS with transitions of the form $\Delta \xrightarrow{a} \Delta'$:
- Defining Petri nets as $\mathbb{N}$-ULTRAS models in which states are Petri net places and transitions are Petri net transitions.

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