# Reversibility and Concurrency

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# Overview

# Outline

# Reversibility

Reversibility is very common in physics and biochemistry.
In nature reversibility underpins many mechanisms for achieving progress or change.

- e.g. catalysis, building polymers

In artificial systems reversibility has a growing number of applications:

- saving energy
- recovery from failure
  e.g. long-running transactions with compensations
- debugging

# Forms of reversibility

Causal order reversing: events are undone (written $\underline{e}$) preserving causal order

- transaction with compensation: $s_1 < s_2 < s_3$ and $c$.
  Pattern of behaviour: $s_1\ s_2\ \underline{s_2}\ \underline{s_1}\ c$

Out-of-causal order reversing: patterns of undoing of events appear to violate causality

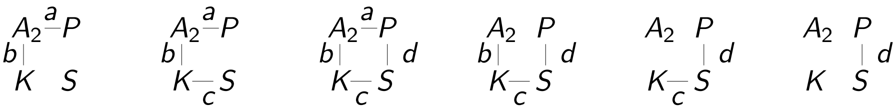
- industrial plant robots
- biochemistry
  e.g. catalysis: $a\ b\ \underline{a}$
  
  ERK signalling pathway: $a\ b\ \underline{a}\ c\ \underline{b}\ \underline{c}$

# Example

Basic catalytic cycle for protein substrate phosphorylation by a kinase.

- Adenosine DiPhosphate (ADP) $A_2$, Adenosine TriPhospate (ATP) is $A_2 - P$
- Kinase $K$ - the catalyst, substrate $S$, phosphate $P$

$P$ is transferred from $A_2 - P$ to $S$.




Behaviour: $a\ b\ c\ d\ \underline{a}\ \underline{b}\ \underline{c}$

The order in which bonds are created and broken varies in such catalytic cycles; we wish to allow reversing events in an arbitrary order.

# Reachable states

We study reachable states.

The most interesting are reachable states that are not forwards reachable.

Very common in mechanisms in nature that deliver change or progress while taking care of deadlock and failure.

How to model out-of-causal order (general) reversibility with

- event structures
- process calculi

# Outline

# Prime event structures

(Nielsen, Plotkin & Winskel)

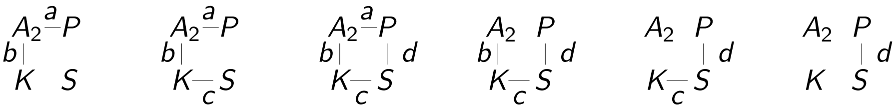
Prime event structures (PES) are triples $(E, <, \sharp)$ where

- $E$ set of events, ranged over by $e, a, b$
- causation $a < b$ (transitive)
- conflict $a \sharp b$ (symmetric)

configurations $X$

- sets of events that have happened so far
- initially $\emptyset$
- conflict-free

# Modelling




- Let events $a, b, c, d$ represent (creation of) the bonds $a, b, c, d$.
- $a < b < c < d$
- undoing of $a, b, c$ (breaking bonds) represented by $\underline{a}, \underline{b}, \underline{c}$
- reversing an event $a$ means that $a$ is removed from the current configuration

# Reverse causation and prevention

CONCUR 2013: Add to PES a new reverse causality relation $\prec$:

- $d \prec \underline{a}$, $d \prec \underline{b}$, $d \prec \underline{c}$ - need $d$ to undo $a, b, c$
- also $a \prec \underline{a}$, $b \prec \underline{b}$ and $c \prec \underline{c}$

We do not include $d \prec \underline{d}$, since $d$ is irreversible here.

Extend PES further with a prevention relation $\triangleright$:

- $a \triangleright \underline{b}$ prevents undoing of $b$ while $a$ is present
- similarly $b \triangleright \underline{c}$

Get the desired ordering of $\underline{a}, \underline{b}, \underline{c}$.

Then $(\{a, b, c, d\}, \{a, b, c\}, <, \sharp, \prec, \triangleright)$ (with empty conflict $\sharp$) is a Reversible PES (RPES).

# Asymmetric event structures

Asymmetric ESs $(E, <, \lhd)$ (Baldan, Corradini & Montanari) :
Like PESs, except that symmetric conflict $\sharp$ replaced by asymmetric conflict (precedence) $\lhd$.

We write $a \lhd b$ iff $b \rhd a$.

Dual interpretation:

- $a \lhd b$ says that $a$ precedes event $b$, meaning that if both $a$ and $b$ occur then $a$ occurred first
- $b \rhd a$ says that $b$ prevents $a$, meaning that if $b$ is present in a configuration then $a$ cannot occur.

We have already used prevention $b \rhd \underline{a}$ on reverse events with RPESs.
$a \lhd b$ will give us greater control of forward events in the reversible setting.

# Reversible Asymmetric ESs

We generalise RPESs to the setting of asymmetric conflict $\lhd$ and not necessarily transitive causation $\prec$.

A reversible asymmetric event structure (RAES) is $(E, F, \prec, \lhd)$ where

- $\prec \subseteq E \times (E \cup \underline{F})$ is the direct causation relation, which combines forwards causation $<$ and reverse causation $\prec$ of RPESs
- $\lhd \subseteq (E \cup \underline{F}) \times E$ is the precedence relation, which combines forwards precedence $\lhd$ of AESs and reverse prevention $\rhd$ of RPESs

# Configuration systems

A configuration system is $(E, F, \mathsf{C}, \stackrel{A \cup \underline{B}}{\rightarrow})$ where $\mathsf{C}$ is the set of configurations.

Concurrent enabling: if $X \stackrel{A \cup \underline{B}}{\rightarrow} Y$ then all possible splits into sub-steps are enabled. For example, if $\{a\} \stackrel{\{b, \underline{a}\}}{\rightarrow} \{b\}$ then

$$\{a\} \stackrel{b}{\rightarrow} \{a, b\} \stackrel{\underline{a}}{\rightarrow} \{b\} \text{ and } \{a\} \stackrel{\underline{a}}{\rightarrow} \emptyset \stackrel{b}{\rightarrow} \{b\}$$

# Reachable states that are not forwards reachable

Note that if $Y = X \cup \{a\}$ and $X, Y \in \mathsf{C}$ then usually $X \to Y$.

This may no longer hold in the reversible setting.

As an example, let $E = \{a, b\}$ with $a < b$.
Then $\{b\}$ is not a possible configuration using forwards computation.
However if $a$ is reversible:

$$\emptyset \xrightarrow{a} \{a\} \xrightarrow{b} \{a, b\} \xrightarrow{\underline{a}} \{b\}$$

Thus both $\emptyset$ and $\{b\}$ are configurations, but we do not have $\emptyset \xrightarrow{b} \{b\}$.

# Event structures of van Glabbeek & Plotkin

$(E, \mathsf{Con}, X \vdash Y)$: very general, correspond to 1-occurrence Petri nets.
ES $\supset$ AES $\supset$ PES.

In RC 2013 we proposed a new enabling $\vdash'$ to capture general reversibility

- $X \oslash Y \vdash' e$: events in $Y$ prevent $e$
- $X \oslash Y \vdash' \underline{e}$

Obtain reversible ESs (RESs) $(E, F, \mathsf{Con}, \vdash')$, and RES $\supset$ RAES $\supset$ RPES.

Questions:

- relationship between $\vdash$ and $\vdash'$
- have $\xrightarrow{e}$ for RES. How to define $\xrightarrow{A}$ or even $\xrightarrow{A \cup \underline{B}}$?

# Outline

# CCS with communication keys

Need memory to execute processes (forwards and) in reverse.

$\nu a\,(b.a \mid \overline{a}.c) \stackrel{b[1]}{\rightarrow} \nu a\,(b[1].a \mid \overline{a}.c) \stackrel{\tau[2]}{\rightarrow} \nu a\,(b[1].a[2] \mid \overline{a[2]}.c)$

- $\stackrel{\tau[2]}{\rightarrow} \nu a\,(b[1].a \mid \overline{a}.c) \stackrel{b[1]}{\rightarrow}$
- $\stackrel{c[4]}{\rightarrow} \nu a\,(b[1].a[2] \mid \overline{a[2]}.c[4]) \stackrel{\tau[2]}{\not\rightarrow}$

Current reversible calculi

- CCSK and RCCS (Danos, Krivine)
- roll-$\pi$ (Lanese, Mezzina, Stefani)
- reversible $\pi$ (Cristescu, Krivine, Varacca)

suitable for causal-order reversing but unable to model general reversibility.

# Towards calculi for general reversibility

1. CCSK with controller processes (RC 2012)
   - multiset prefixing
   - controllers determine direction and pattern of computation
2. A calculus for reversible event structures: no prefixing, constructs for forwards and reverse causality
3. In CCSK $a.b[1].c$ makes no sense. Could be seen as implementing a pattern of desired behaviour:

$$\downarrow a.b[1].c \stackrel{a[2]}{\rightarrow} a[2].\downarrow b[1].c \stackrel{b[1]}{\Rightarrow} a[2].b.\downarrow c \stackrel{c[5]}{\rightarrow} a[2].b.c[5]\downarrow$$

Also, for example, $a[2].\downarrow b[1].c \stackrel{a[2]}{\Rightarrow} \downarrow a.b[1].c$.

# Outline

# Conclusions

Motivated the need for the modelling of out-of-causal order reversibility.

- Reversible PESs and AESs are suitable for out-of-causal order reversing.
- Pointers towards general reversible ESs.
- Reversible calculi not well suited for out-of-causal order reversing. Presented some ideas for more appropriate calculi.

# References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