On the Relative Expressiveness of Process Calculi

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Goal

To order system description languages like CCS, CSP and the π-calculus w.r.t. their expressive power.

e.g.

ACP
$\uparrow$
π-calculus
$\uparrow$
join calculus
$\uparrow$
asynchronous π-calculus

Language A is at least as expressive as language B if there exists a correct translation or valid encoding of B into A.

Hence the first goal, pursued in this talk, is to propose a good notion of *correct translation* or *valid encoding*.

Prior work (< 2000)

I deal with languages where operations and recursion constructs are interpreted as operations of on a set of values, called a *domain*.

- First concept of [correct] translation between such languages in BOUDOL 1985.
- Formulation of essentially same idea in VAN GLABBEEK 1994.

In both works (and also here) the translation is *up to* a semantic equivalence $\sim$. A stronger ($=$ finer, or more discriminating) equivalence gives a stronger encodability result, or a weaker separation result.

Limitations:

- [Bo85] and [vG85] require $\sim$ to be a congruence for the source as well as the target language.
- • [Bo85] requires the domain in which a language $\mathcal{L}$ is interpreted to consist of the closed expressions of $\mathcal{L}$.
  • [vG94] has the weaker requirement that all (relevant) elements in the domain are denotable by closed terms.

Overview of main results — Part I

Limitations of existing notion of a correct translation between two languages:

- Congruence: [Bo85] and [vG85] require $\sim$ to be a congruence for the source as well as the target language.
- Denotablity: all (relevant) elements in the domain are denotable by closed terms.

This work lifts these limitations.

- I propose a concept of a *correct* translation that lifts the denotability requirement, and merely requires $\sim$ to be a congruence for the source language, and for the source's image within the target. This is written down in [vG12].
- I also propose a concept of a *valid* translation that lifts both. This generalises the notion of validity of [vG12], that lifted the congruence requirement, but kept denotability.

The notions agree in situations where both apply.

Correct translations

I represent a language $\mathcal{L}$ as a pair $(\mathbb{T}_\mathcal{L}, [\![\ ]\!]_\mathcal{L})$ of a set $\mathbb{T}_\mathcal{L}$ of terms and a mapping $[\![\ ]\!]_\mathcal{L} : \mathbb{T}_\mathcal{L} \to \mathcal{D}_\mathcal{L}$ onto a set of meanings.

A *translation* from $\mathcal{L}$ into $\mathcal{L}'$ is a mapping $\mathcal{T} : \mathbb{T}_\mathcal{L} \to \mathbb{T}_{\mathcal{L}'}$.
It is *correct* when $[\![\mathcal{T}(E)]\!]_{\mathcal{L}'} = [\![E]\!]_\mathcal{L}$ for all $E \in \mathbb{T}_\mathcal{L}$.

Language $\mathcal{L}'$ is at least as *expressive* as $\mathcal{L}$ if $\exists$ correct translation.

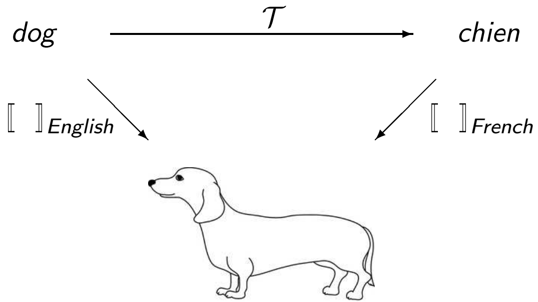

Figure: The essence of a correct translation

A correct translation from $\mathcal{L}$ to $\mathcal{L}'$ exists iff $\mathcal{D}_\mathcal{L} \subseteq \mathcal{D}_{\mathcal{L}'}$.

Dividing out a semantic equivalence

A *translation* from $\mathcal{L}$ into $\mathcal{L}'$ is a mapping $\mathcal{T} : \mathbb{T}_{\mathcal{L}} \to \mathbb{T}_{\mathcal{L}'}$.
It is *correct* when $[\![\mathcal{T}(E)]\!]_{\mathcal{L}'} = [\![E]\!]_{\mathcal{L}}$ for all $E \in \mathbb{T}_{\mathcal{L}}$.

It is *correct up to* $\sim$ when $[\![\mathcal{T}(E)]\!]_{\mathcal{L}'} \sim [\![E]\!]_{\mathcal{L}}$ for all $E \in \mathbb{T}_{\mathcal{L}}$.

A correct translation up to $\sim$ is the same as a correct translation into the quotient domain of $\sim$-equivalence classes of values.

Translating operators

From here on I deal with languages that have *variables*, *operators* and possibly *recursion constructs*, that are interpreted in a domain of values **D**.

The semantic mapping is of type

$$[\![\ ]\!]_{\mathcal{L}} : \mathbb{T}_{\mathcal{L}} \rightarrow ((\mathcal{X} \rightarrow \mathbf{D}) \rightarrow \mathbf{D})$$

so that $[\![E]\!]_{\mathcal{L}}$ is a value from **D** that depends on the choice of a *valuation* $\rho : \mathcal{X} \rightarrow \mathbf{D}$ of the variables that occur in expression E.

A translation $\mathcal{T} : \mathbb{T}_{\mathcal{L}} \rightarrow \mathbb{T}_{\mathcal{L}'}$ is correct iff $\mathbf{D} \subseteq \mathbf{D}'$ and

$$[\![\mathcal{T}(E)]\!]_{\mathcal{L}'}(\rho) = [\![E]\!]_{\mathcal{L}}(\rho)$$

for all $E \in \mathbb{T}_{\mathcal{L}}$ and all valuations $\rho : \mathcal{X} \rightarrow \mathbf{D}$.

Example

Let $\mathcal{L}$ be the language whose syntax is the operator $+$, interpreted as addition on the domain of the natural numbers.

$\mathcal{L}'$ is the language with operators e^x and $\ln(x)$ on the reals, as well as multiplication $\times$.

Using that $\ln(e^x) = x$, the $\mathcal{L}$-expression $X + Y$ can be translated into to $\mathcal{L}'$-expression $\ln(e^X \times e^Y)$.

Using this, a translation $\mathcal{T} : \mathbb{T}_{\mathcal{L}} \to \mathbb{T}_{\mathcal{L}'}$ is defined inductively by $\mathcal{T}(X) = X$ and $\mathcal{T}(E + F) = \ln(e^{\mathcal{T}(E)} \times e^{\mathcal{T}(F)})$.

Correct translations up to a semantic equivalence

Consider languages $\mathcal{L}$ and $\mathcal{L}'$ with semantic mappings

$$[\![\ ]\!]_{\mathcal{L}} : \mathbb{T}_{\mathcal{L}} \to ((\mathcal{X} \to \mathbf{V}) \to \mathbf{V})$$

$$[\![\ ]\!]_{\mathcal{L}'} : \mathbb{T}_{\mathcal{L}'} \to ((\mathcal{X} \to \mathbf{V}') \to \mathbf{V}')$$

Let $\sim$ be an equivalence on some $\mathbf{Z} \supseteq \mathbf{V}, \mathbf{V}'$.

For two valuations $\eta, \rho : \mathcal{X} \to \mathbf{Z}$, write $\eta \sim \rho$ if $\eta(X) \sim \rho(X)$ for each $X \in \mathcal{X}$.

A translation $\mathcal{T} : \mathbb{T}_{\mathcal{L}} \to \mathbb{T}_{\mathcal{L}'}$ is *correct up to* $\sim$ if

- $\forall v \in \mathbf{V}.\ \exists v' \in \mathbf{V}'.\ v' \sim v$, and
- $[\![\mathcal{T}(E)]\!]_{\mathcal{L}'}(\eta) \sim [\![E]\!]_{\mathcal{L}}(\rho)$ for all $E \in \mathbb{T}_{\mathcal{L}}$ and all valuations $\eta : \mathcal{X} \to \mathbf{V}'$ and $\rho : \mathcal{X} \to \mathbf{V}$ with $\eta \sim \rho$.

This definition can be seen as an instantiation of my original notion of a correct translation.

Correctness implies congruence

An equivalence $\sim$ is a *congruence* for $\mathcal{L}$ if $[\![E]\!]_{\mathcal{L}}(\nu) \sim [\![E]\!]_{\mathcal{L}}(\rho)$ for all $E \in \mathbb{T}_{\mathcal{L}}$ and all valuations $\nu, \rho : \mathcal{X} \to \mathbf{V}$ with $\nu \sim \rho$.

Proposition: If $\mathcal{T}$ is a correct translation up to $\sim$ from $\mathcal{L}$ to $\mathcal{L}'$ then $\sim$ is a congruence from $\mathcal{L}$ as well as for $\mathcal{T}(\mathcal{L})$.

$\sim$ is a *congruence* for $\mathcal{T}(\mathcal{L})$ if $[\![\mathcal{T}(E)]\!]_{\mathcal{L}'}(\nu) \sim [\![\mathcal{T}(E)]\!]_{\mathcal{L}'}(\rho)$ for all $E \in \mathbb{T}_{\mathcal{L}}$ and all valuations $\nu, \rho : \mathcal{X} \to \mathbf{U}$ with $\nu \sim \rho$.

Here $\mathbf{U} := \{v' \in \mathbf{V}' \mid \exists v \in \mathbf{V}.\ v' \sim v\}$.

Applications

As illustration of the theory, I showed that there is a correct translation up to trace equivalence of CSP into CCS, but not up to convergent weak bisimilarity [vG12].

There is also no correct translation up to weak bisimulation congruence of the $+$ of CCS into CSP.

In [dS85] and [vG93] correct translations up to strong bisimilarity are presented that encode any De Simone language into $\textsc{Meije}$, SCCS or ACP.

Semantic equivalences for comparing wildly different languages

To compare the expressive power of, say, CCS and the λ-calculus, weak bisimulation is not the right semantic equivalence.

An a-transition of a CCS-expression can never be matched by any transition of the λ-calculus, as the latter doesn't do a-transitions.

However, both languages have a *reduction semantics*, that provides an unlabelled transition relation $E \longmapsto F$.

For CCS, the $\longmapsto$-relation consists of exactly the τ-transitions in the labelled transition system semantics,

Hence, to compare these languages we can use forms of bisimulation that ignore all a-transitions, and deal with τ-transitions, or, equivalently, reductions, only.

Bisimulations that only count τ-transitions

Definition: A symmetric binary relation $\mathcal{R}$ between processes is a *reduction bisimulation* if it satisfies:

- if $P\mathcal{R}Q$ and $P \xrightarrow{\tau} P'$ then $\exists Q'$ with $Q \xrightarrow{\tau} Q'$ and $P'\mathcal{R}Q'$.

Two processes are *reduction bisimilar* if they are related by a reduction bisimulation.

Now one can compare the expressive power of the λ-calculus and CCS up to reduction bisimilarity.

However, reduction bisimilarity is a not a congruence for CCS, so my theory of correct translations does not apply here.

Valid translations up to a preorder

Definition: A *semantic translation* from **V** into **V′** is a relation $\mathbf{R} \subseteq \mathbf{V}' \times \mathbf{V}$ such that $\forall v \in \mathbf{V}.\, \exists v' \in \mathbf{V}'.\, v' \;\mathbf{R}\; v$.

For $\eta{:}\mathcal{X} \rightarrow \mathbf{V}'$, $\rho{:}\mathcal{X} \rightarrow \mathbf{V}$ write $\eta \;\mathbf{R}\; \rho$ iff $\eta(X) \;\mathbf{R}\; \rho(X)$ for each X.

$\mathcal{T} : \mathbb{T}_{\mathcal{L}} \rightarrow \mathbb{T}_{\mathcal{L}'}$ is *correct* w.r.t. **R** if $[\![\mathcal{T}(E)]\!]_{\mathcal{L}'}(\eta) \;\mathbf{R}\; [\![E]\!]_{\mathcal{L}}(\rho)$ for all E and all η and ρ with $\eta \;\mathbf{R}\; \rho$.

Theorem: $\mathcal{T}$ is correct up to $\sim$ iff $\sim$ restricted to $\mathbf{V}' \times \mathbf{V}$ is a semantic translation and $\mathcal{T}$ is correct w.r.t. **R**.

Definition: A translation $\mathcal{T}$ is *valid* up to $\bullet\!\!\sim$ iff it is correct w.r.t. some semantic translation $\mathbf{R} \subseteq \bullet\!\!\sim$.

Theorem: $\mathcal{T}$ is correct up to $\sim$
iff $\mathcal{T}$ is valid up to $\sim$ and $\sim$ is a congruence for $\mathcal{T}(\mathcal{L})$.

Definition: Language $\mathcal{L}'$ is at least as *expressive* as $\mathcal{L}$ up to $\bullet\!\!\sim$ if a valid translation from $\mathcal{L}$ to $\mathcal{L}'$ exists.

A hierarchy of expressiveness preorders

Let $\bullet\!\!\sim$ be any semantic equivalence (or preorder).

Proposition: The identity is a valid translation up to $\bullet\!\!\sim$ from any language into itself.

Theorem: If valid translations up to $\bullet\!\!\sim$ exists from $\mathcal{L}_1$ into $\mathcal{L}_2$ and from $\mathcal{L}_2$ into $\mathcal{L}_3$, then there is one from $\mathcal{L}_1$ into $\mathcal{L}_3$.

So "being at least as expressive up to $\bullet\!\!\sim$" is a preorder.

In fact, there is a category with the languages as the objects and the valid translations as the morphisms.

Theorem: Let $\mathcal{T}$ be a translation from $\mathcal{L}$ to $\mathcal{L}'$, and let $\sim, \approx$ be semantic equivalences (or preorders) for $\mathcal{T}(\mathcal{L})$ with

$$p \sim q \Rightarrow p \approx q.$$

If $\mathcal{T}$ is valid up to $\sim$, then it is valid up to $\approx$.

Compositionality

Definition: A translation $\mathcal{T}$ from $\mathcal{L}$ into $\mathcal{L}'$ is *compositional* if

$$\mathcal{T}(E[\sigma]) \stackrel{\alpha}{=} \mathcal{T}(E)[\mathcal{T} \circ \sigma]$$

for each expression E and substitution σ, and also $\mathcal{T}(X) = X$.

In case $E = f(t_1, \ldots, t_n)$ this amounts to

$$\mathcal{T}(f(t_1, \ldots, t_n)) \stackrel{\alpha}{=} E_f(\mathcal{T}(t_1), \ldots, \mathcal{T}(t_n)),$$

where $E_f := \mathcal{T}(f(X_1, \ldots, X_n))$.

Theorem: If any valid translation up to $\bullet\!\!\sim$ from $\mathcal{L}$ to $\mathcal{L}'$ exists, then there is a compositional one.

Bisimulations that only count τ-transitions

Definition: A symmetric binary relation $\mathcal{R}$ between processes is a *reduction bisimulation* if it satisfies:

- if $P\mathcal{R}Q$ and $P \xrightarrow{\tau} P'$ then $\exists Q'$ with $Q \xrightarrow{\tau} Q'$ and $P'\mathcal{R}Q'$.

Two processes are *reduction bisimilar* if they are related by a reduction bisimulation.

Now one can compare the expressive power of the λ-calculus and CCS up to reduction bisimilarity.

Indirectly, such an equivalence says also something about the transitions that are not labelled τ. For instance $a.0$ is a process that cannot do any τ's. However, when put in parallel with $\bar{a}.0$ it can produce a τ-transition. So if $\mathcal{T}$ is a valid translation from CCS into the λ-calculus, by compositionality it should satisfy

$$\mathcal{T}(a.0|\bar{a}.0) = C_|(\mathcal{T}(a.0)|\mathcal{T}(\bar{a}.0)).$$

This shows, that it won't do to simply translate the CCS expression $a.0$ into (the λ-calculus equivalent of) the process 0.

Connecting valid and correct transitions

Theorem: If a translation between two languages is valid up to an equivalence $\sim$, then it is even valid (and in fact correct) up to an equivalence whose restriction to the source language is the congruence closure of $\sim$.

Application: Suppose we have a translation from finite CCS into the λ-calculus that is valid up to reduction equivalence, then we know it is even correct up to the congruence close of reduction equivalence. Within finite CCS that congruence closure equals strong bisimilarity; within the λ-calculus it may have a completely different form . . .

Barbed bisimulation

For general CCS the congruence close of reduction bisimilarity is not quite strong bisimilarity, and weak reduction bisimilarity turns out to be the universal relation.

These notions must be upgraded a minimal amount, so as to be able to report that a system is in a success state (a la Hennessy-De Nicola-testing).

This gives rise to strong and weak *barbed* bisimulation, a notion that can be applied to the λ-calculus just as well as to CCS.

Related work

A great number of encodability and separation results have been published recently. Many employ different and somewhat ad-hoc criteria on what is a valid encoding.

Gorla collected some essential features and integrated them into a proposal for a valid encoding.

- Gorla uses a weaker notion of compositionality than me, where the expression E_f encoding an operator f may be dependent in the *names* occurring freely in the arguments of f.
  Question: Is this needed?
  Are there good encodings that satisfy this but fail true compositionality?
- Gorla requires a form of invariance under name-substitution. I do not. Even if this would be a good property, Gorla's version is too restrictive, as it invalidates a token example of a good encoding: of the input process $a(x).E$ of value-passing CCS into the CCS expression $\sum_{v\in\mathcal{V}} a_v.E[v/x]$, where $\mathcal{V}$ is given set of data values.

Gorla's concept of valid translation

- The remaining three criteria of Gorla are *semantic*: they come close to singling out a particular preorder for comparing terms and their translations.

Let *AG-valid* be Gorla's notion of validity, amended in two ways: skip the second requirement and strengthen the first to full compositionality.

Conjecture: AG-validity is slightly weaker than my notion of validity up to weak barbed bisimilarity with explicit divergence, and slightly stronger than my notion of validity up to barbed coupled simulation.

Question: I'm curious which of the existent encoding and separability results fit in my framework, and how dependent they are on the chosen equivalence or preorder.

Application: Comparing CCS and the π-calculus

Many separation results in the literature are based on the assumption that parallel composition translates homomorphically: $\mathcal{T}(E|F) = \mathcal{T}(E)|\mathcal{T}(F)$.

This is often defended by the theory that non-homomorphic translations reduce the degree of concurrency of the source process — a theory I do not share.

The proofs of Palamidessi and of of Phillips et al. that there is no encoding of π_a into CCS uses this assumption. Gorla's alternative proof relaxes this assumption but uses his too restrictive second criterion.

Hence the question whether π_a is expressible in CCS is still wide open.

Application: Comparing CCS and the π-calculus

I can show that there is no translation valid up to reduction equivalence.

This contradicts a result of Banach & van Breugel [in a Technical report labelled "draft"] that presents such a translation.

However, there is a translation, valid under strong barbed bisimulation, of π_a into CCS upgraded with the communication function of ACP.