Bridging the Gap Between Binary and Multiparty Communications

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Open Problems in Concurrency Theory (OPCT)
Bertinoro, June 2014

Outline

Large-Scale Software Infrastructures

- Massive collections of heterogeneous, communicating services
- Correctness is a combination of several issues, including:
  - Resource usage policies
  - Security and trustworthiness requirements
  - Conformance to predefined protocols

Large-Scale Software Infrastructures: Protocols

- Rely on advanced forms of mobility, concurrency, and distribution
- Conveniently described as chroreographies
  - ⋆ A global description of the overall interactive scenario
  - ⋆ Descriptions of the local behavior for each participant
  - ⋆ Ways of ensuring that implementations "respect" global and local descriptions.
- Several analysis techniques proposed, including:
  - ⋆ Models/standards for (semi)formal description/analysis (e.g., BPMN)
  - ⋆ Automata-based approaches (e.g., MSCs/MSGs, CFSMs)
  - ⋆ Type-based approaches, such as session types

Session Types: A Class of Behavioral Types

Seminal approach to the analysis structured communications
[Honda (1993); Honda, Vasconcelos, Kubo (1998)]

- Communication protocols structured into sessions
- Concurrent processes communicating through session channels
- Disciplined interactive behavior, abstracted as **session types**

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Correctness guarantees for specifications:

- Adhere to their ascribed session protocols - Fidelity
- Do not feature runtime errors – Safety
- Do not get stuck – Progress / Lock-Freedom
- Do not have infinite reduction sequences – Termination

STs for Multiparty Communications

- Multiparty Session Types (MPSTs) [Honda, Yoshida, Carbone (2008)]
  - ⋆ Protocols may involve more than two partners
  - ⋆ Global and local types, related by a projection function
  - ⋆ Underlying theory is subtle; analysis techniques hard to obtain

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  - ⋆ Well-understood theory and analysis techniques

  Foundational significance:

  Linear logic propositions as session types, in the style of Curry-Howard [Caires and Pfenning (2010); Wadler (2012)]

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- A reduction would be
  - ⋆ theoretically insightful
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- Practice suggests that MPSTs are more expressive than BSTs
- Open problem: We don't know of any formal results

Outline

This Talk: A Positive Result

We present a formal, two-way correspondence between

- MPSTs with labeled communication and parallel composition, following [Honda, Yoshida, Carbone (2008), Deniélou and Yoshida (2013)]
- BSTs based on linear logic, following [Caires and Pfenning (2010)]: fidelity, safety, termination, and (dead)lock-freedom by typing.

Our Approach

- We decouple a multiparty communication from $\mathtt{p}$ to $\mathtt{q}$:
  - ⋆ A send action from $\mathtt{p}$ to some intermediate entity
  - ⋆ A forwarding action from the entity to $\mathtt{q}$

Our Approach: Medium Processes

- We decouple a multiparty communication from $\mathtt{p}$ to $\mathtt{q}$:
  - ⋆ A send action from $\mathtt{p}$ to some intermediate entity
  - ⋆ A forwarding action from the entity to $\mathtt{q}$
- Given a global type G, extract its medium process $\mathsf{M}[\![G]\!]$
  - ⋆ Intermediate party in all multiparty exchanges
  - ⋆ Captures sequencing information in G by decoupling interactions
  - ⋆ Local implementations need not know about the medium

MPSTs and BSTs: A Two-Way Correspondence

1. Let G be a well-formed global type.
   $\mathsf{M}[\![G]\!]$ is well-typed under an environment in which participants are assigned types corresponding to the projections of G.

2. Let $\mathsf{M}[\![G]\!]$ be a well-typed medium process under an environment in which participants are assigned some binary types. Such binary types correspond, in a precise sense, to the projections of G.

A Possible Methodology

Revising the one proposed in [Honda, Yoshida, Carbone (2008)]

(i) A developer describes an intended interaction scenario as a global type G.

(ii) She extracts $\mathsf{M}[\![G]\!]$ and the set of (local) binary session types representing the projection of G onto all participants.

(iii) Using logic-based BSTs she checks that $\mathsf{M}[\![G]\!]$ is well-typed with respect to the set of (local) binary types just extracted. This ensures deadlock-freedom.

(iv) She develops code, one for each participant, validating its conformance to the corresponding (local) session type.

Two Different Worlds, Connected via Mediums

- Multiparty interactions now explained from two different angles
- Half-way between two essentially distinct, foundational theories
- Clean justifications, based on linear logic, for MPSTs concepts:
  - semantics of global types
  - definitions of projection/well-formedness
- Naturally handles name passing, delegation, parallel composition
- Direct connection from choreographies to process implementations
- Techniques for binary processes applicable on global specifications:
  - Deadlock freedom
  - Typed behavioral equivalences

Outline

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A Standard Session π-calculus

- Given names $(x, y, z, \ldots)$, processes (P, Q, R) are defined by

$$\begin{array}{rlcl} P ::= & \mathbf{0} & \mid & P \mid Q & \mid & (\boldsymbol{\nu} y)P \\ \mid & \overline{x}\, y.P & \mid & x(y).P & \mid & !x(y).P \\ \mid & x \triangleleft \mathtt{l}_i; P & \mid & x \triangleright \{\mathtt{l}_i : P_i\}_{i \in I} & \mid & [x \leftrightarrow y] \end{array}$$

- We write $\overline{x}(y)$ to stand for the bound output $(\boldsymbol{\nu} y)\overline{x}\, y$.
- An associated LTS with expected labels:

$$\lambda \quad ::= \quad \tau \mid x(y) \mid x \triangleleft \mathtt{l} \mid \overline{x}\, y \mid \overline{x}(y) \mid \overline{x \triangleleft \mathtt{l}}$$

MPSTs: Syntax

- The language of global types subsumes those given in [Honda, Yoshida, Carbone (2008), Deniélou and Yoshida (2013)]

- Define global and local types as

$$\begin{array}{rcl} G & ::= & \mathsf{end} \mid \mathtt{p} \twoheadrightarrow \mathtt{q}{:}\{\mathtt{l}_i\langle U_i\rangle.G_i\}_{i\in I} \mid G_1 \mid G_2 \\ T & ::= & \mathsf{end} \mid \mathtt{p}?\{\mathtt{l}_i\langle U_i\rangle.T_i\}_{i\in I} \mid \mathtt{p}!\{\mathtt{l}_i\langle U_i\rangle.T_i\}_{i\in I} \\ U & ::= & \mathsf{bool} \mid \mathsf{nat} \mid \mathsf{str} \mid \ldots \mid T \end{array}$$

- $G \upharpoonright \mathtt{p}_i$ is the (merge-based) projection of G onto participant $\mathtt{p}_i$

- Well-formedness of G is defined as correct projectability on all $\mathtt{p}_i$

Choreographies as MPSTs: A Commit Protocol

Structured interaction among three participants $\mathtt{p_A}$, $\mathtt{p_B}$, and $\mathtt{p_C}$:

$$
\begin{aligned}
G = \mathtt{p_A} \twoheadrightarrow \mathtt{p_B} : \big\{ & \mathtt{act}\langle \mathsf{int} \rangle . \\
& \qquad \mathtt{p_B} \twoheadrightarrow \mathtt{p_C} : \big\{ \mathtt{sig}\langle \mathsf{str} \rangle . \\
& \qquad\qquad \mathtt{p_A} \twoheadrightarrow \mathtt{p_C} : \{ \mathtt{comm}\langle \mathbf{1} \rangle . \mathsf{end} \} \big\} , \\
& \mathtt{quit}\langle \mathsf{int} \rangle . \\
& \qquad \mathtt{p_B} \twoheadrightarrow \mathtt{p_C} : \big\{ \mathtt{save}\langle \mathbf{1} \rangle . \\
& \qquad\qquad \mathtt{p_A} \twoheadrightarrow \mathtt{p_C} : \{ \mathtt{fin}\langle \mathbf{1} \rangle . \mathsf{end} \} \ \big\} \ \big\}
\end{aligned}
$$

The projections of G onto $\mathtt{p_A}$ and $\mathtt{p_C}$:

$$
\begin{aligned}
G \!\upharpoonright\! \mathtt{p_A} = \ & \mathtt{p_A} ! \big\{ \mathtt{act}\langle \mathsf{int} \rangle . \mathtt{p_A} ! \{ \mathtt{comm}\langle \mathbf{1} \rangle . \mathsf{end} \} , \\
& \qquad \mathtt{quit}\langle \mathsf{int} \rangle . \mathtt{p_B} ! \{ \mathtt{sig}\langle \mathsf{str} \rangle . \mathsf{end} \} \ \big\} \\
G \!\upharpoonright\! \mathtt{p_C} = \ & \mathtt{p_B} ? \big\{ \mathtt{sig}\langle \mathsf{str} \rangle . \mathtt{p_A} ? \{ \mathtt{comm}\langle \mathbf{1} \rangle . \mathsf{end} \} , \\
& \qquad \mathtt{save}\langle \mathbf{1} \rangle . \mathtt{p_A} ? \{ \mathtt{fin}\langle \mathbf{1} \rangle . \mathsf{end} \} \ \big\}
\end{aligned}
$$

Outline

Medium Process of a Global Type

The medium of global type G, noted $\mathsf{M}[\![G]\!]$, is defined inductively as:

- $\mathsf{M}[\![\mathtt{end}]\!] = \mathbf{0}$
- $\mathsf{M}[\![\, \mathtt{p} \twoheadrightarrow \mathtt{q}{:}\{\mathtt{l}_i\langle U_i\rangle.G_i\}_{i\in I} \,]\!] =$
$$c_{\mathtt{p}} \triangleright \big\{\mathtt{l}_i : c_{\mathtt{p}}(u).c_{\mathtt{q}} \triangleleft \mathtt{l}_i; \overline{c_{\mathtt{q}}}(v).([u \leftrightarrow v] \mid \mathsf{M}[\![G_i]\!])\big\}_{i\in I}$$
- $\mathsf{M}[\![G_1 \mid G_2]\!] = \mathsf{M}[\![G_1]\!] \mid \mathsf{M}[\![G_2]\!]$

Correspondence Between G and $\mathsf{M}[\![G]\!]$ (Informal)

Let $G = \mathtt{p} \twoheadrightarrow \mathtt{q}{:}\{\mathtt{l}_i\langle U_i\rangle.G_i\}_{i\in I}$. Then, we have:

$$\mathsf{M}[\![G]\!] \quad = \quad c_{\mathtt{p}} \triangleright \big\{\mathtt{l}_i : c_{\mathtt{p}}(u).c_{\mathtt{q}} \triangleleft \mathtt{l}_i; \overline{c_{\mathtt{q}}}(v).([u \leftrightarrow v] \mid \mathsf{M}[\![G_i]\!])\big\}_{i\in I}$$

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& \xrightarrow{c_{\mathtt{p}}(y)} & c_{\mathtt{q}} \triangleleft \mathtt{l}_j; \overline{c_{\mathtt{q}}}(v).([y \leftrightarrow v] \mid \mathsf{M}[\![G_j]\!])
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\end{array}
$$

An Example: The Commit Protocol

$$G = \mathtt{p_A} \twoheadrightarrow \mathtt{p_B}{:}\Big\{ \begin{array}{l} \mathtt{act}\langle \mathsf{int}\rangle.\mathtt{p_B} \twoheadrightarrow \mathtt{p_C}{:}\big\{\mathtt{sig}\langle \mathsf{str}\rangle.\mathtt{p_A} \twoheadrightarrow \mathtt{p_C}{:}\{\mathtt{comm}\langle \mathbf{1}\rangle.\mathtt{end}\}\big\}, \\ \mathtt{quit}\langle \mathsf{int}\rangle.\mathtt{p_B} \twoheadrightarrow \mathtt{p_C}{:}\big\{\mathtt{save}\langle \mathbf{1}\rangle.\mathtt{p_A} \twoheadrightarrow \mathtt{p_C}{:}\{\mathtt{fin}\langle \mathbf{1}\rangle.\mathtt{end}\}\big\} \end{array} \Big\}$$

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- The medium process $\mathsf{M}[\![G]\!]$:

$$a \triangleright \big\{ \mathtt{act} : a(v).b \triangleleft \mathtt{act}; \overline{b}(w).([w \leftrightarrow v] \mid b \triangleright \{\mathtt{sig} : b(n).c \triangleleft \mathtt{sig}; \overline{c}(m).([n \leftrightarrow m] \mid a \triangleright \{\mathtt{comm} : a(u).c \triangleleft \mathtt{comm}; \overline{c}(y).([u \leftrightarrow y] \mid \mathbf{0})\}\,)\}) \,,$$
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An Example: The Commit Protocol

- The projections of G – the interface of local implementations:

$$G\restriction \mathsf{p_A} = \mathsf{p_A}!\{\mathtt{act}\langle \mathsf{int}\rangle.\mathsf{p_A}!\{\mathtt{comm}\langle \mathbf{1}\rangle.\mathsf{end}\},\ \mathtt{quit}\langle \mathsf{int}\rangle.\mathsf{p_B}!\{\mathtt{sig}\langle \mathsf{str}\rangle.\mathsf{end}\}\}$$

$$G\restriction \mathsf{p_B} = \mathsf{p_A}?\{\mathtt{act}\langle \mathsf{int}\rangle.\mathsf{p_B}!\{\mathtt{sig}\langle \mathsf{str}\rangle.\mathsf{end}\},\ \mathtt{quit}\langle \mathsf{int}\rangle.\mathsf{p_B}!\{\mathtt{save}\langle \mathbf{1}\rangle.\mathsf{end}\}\}$$

$$G\restriction \mathsf{p_C} = \mathsf{p_B}?\{\mathtt{sig}\langle \mathsf{str}\rangle.\mathsf{p_A}?\{\mathtt{comm}\langle \mathbf{1}\rangle.\mathsf{end}\},\ \mathtt{save}\langle \mathbf{1}\rangle.\mathsf{p_A}?\{\mathtt{fin}\langle \mathbf{1}\rangle.\mathsf{end}\}\}$$

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\end{aligned}
$$

Outline

Correspondence between MPSTs and BSTs

- Conditions under which a medium $\mathsf{M}[\![G]\!]$ is well-typed in the logically motivated BSTs of [Caires & Pfenning (2010)]
- A bidirectional correspondence that relates
  (a) binary session types associated to $\mathsf{M}[\![G]\!]$
  (b) the local types for G

MPSTs and BSTs: Two-Way Correspondence (1)

- The type judgment $\Gamma; \Delta \vdash P :: z{:}C$ (from [Caires & Pfenning (2010)]): P provides behavior C at channel z, building on “services” in $\Gamma; \Delta$

MPSTs and BSTs: Two-Way Correspondence (1)

- The type judgment $\Gamma; \Delta \vdash P :: z{:}C$ (from [Caires & Pfenning (2010)]): P provides behavior C at channel z, building on "services" in $\Gamma; \Delta$
- A compositional typing gives a binary type for all participants.
- Mapping $\langle\langle \cdot \rangle\rangle$ from local types T to binary session types A

Theorem (From Well-Formedness To Typed Mediums)

Let G be a global type, with $\mathsf{part}(G) = \{\mathrm{p}_1, \ldots, \mathrm{p}_n\}$.
If G is well-formed then

$$\Gamma; c_1{:}\langle\langle G \restriction \mathrm{p}_1 \rangle\rangle, \ldots, c_n{:}\langle\langle G \restriction \mathrm{p}_n \rangle\rangle \vdash \mathsf{M}[\![G]\!] :: -{:}\mathbf{1}$$

is a compositional typing for $\mathsf{M}[\![G]\!]$*, for some* Γ.

MPSTs and BSTs: Two-Way Correspondence (2)

- The type judgment $\Gamma; \Delta \vdash P :: z{:}C$ (from [Caires & Pfenning (2010)]): P provides behavior C at channel z, building on "services" in $\Gamma; \Delta$
- A compositional typing gives a binary type for all participants.
- Mapping $\langle\!\langle \cdot \rangle\!\rangle$ from local types T to binary session types A
- Ordering $\preceq^{\sqcup}$ relates local branching types (akin to subtyping)

Theorem (From Well-Typedness To WF Global Types)

Let G be a global type. If

$$\Gamma; c_1{:}A_1, \ldots, c_n{:}A_n \vdash \mathsf{M}[\![G]\!] :: -{:}\mathbf{1}$$

is a compositional typing for $\mathsf{M}[\![G]\!]$ then there exist local types $T_1, \ldots, T_n$ s.t. $G{\restriction}\mathtt{r}_j \preceq^{\sqcup} T_j$ and $\langle\!\langle T_j \rangle\!\rangle = A_j$, for all $\mathtt{r}_j \in G$.

A Behavioral Characterization of Swapping

- The swap relation, written $\simeq_{\mathtt{sw}}$, enables safe transformations over global types [Carbone and Montesi (2013)]. For instance:

$$\frac{\{\mathtt{p}_1, \mathtt{q}_1\} \# \{\mathtt{p}_2, \mathtt{q}_2\}}{\begin{array}{c}\mathtt{p}_1 \twoheadrightarrow \mathtt{q}_1 {:} \big\{ \mathtt{l}_i \langle U_i \rangle . \mathtt{p}_2 \twoheadrightarrow \mathtt{q}_2 {:} \{ \mathtt{l}'_j \langle U'_j \rangle . G_{ij} \}_{j \in J} \big\}_{i \in I} \\ \simeq_{\mathtt{sw}} \\ \mathtt{p}_2 \twoheadrightarrow \mathtt{q}_2 {:} \big\{ \mathtt{l}'_j \langle U'_j \rangle . \mathtt{p}_1 \twoheadrightarrow \mathtt{q}_1 {:} \{ \mathtt{l}_i \langle U_i \rangle . G_{ij} \}_{i \in I} \big\}_{j \in J}\end{array}}$$

A Behavioral Characterization of Swapping

- The swap relation, written $\simeq_{\mathtt{sw}}$, enables safe transformations over global types [Carbone and Montesi (2013)]. For instance:

$$\frac{\{\mathtt{p}_1, \mathtt{q}_1\} \# \{\mathtt{p}_2, \mathtt{q}_2\}}{\begin{array}{c}\mathtt{p}_1 \twoheadrightarrow \mathtt{q}_1 : \Big\{ \mathtt{l}_i \langle U_i \rangle . \mathtt{p}_2 \twoheadrightarrow \mathtt{q}_2 : \{ \mathtt{l}'_j \langle U'_j \rangle . G_{ij} \}_{j \in J} \Big\}_{i \in I} \\ \simeq_{\mathtt{sw}} \\ \mathtt{p}_2 \twoheadrightarrow \mathtt{q}_2 : \Big\{ \mathtt{l}'_j \langle U'_j \rangle . \mathtt{p}_1 \twoheadrightarrow \mathtt{q}_1 : \{ \mathtt{l}_i \langle U_i \rangle . G_{ij} \}_{i \in I} \Big\}_{j \in J}\end{array}}$$

- On logic-based BSTs, we have prefix commutations on processes.

  To justify such transformations, we use context bisimilarity.

  Two typed processes P and Q, are context bisimilar, denoted $\Gamma; \Delta \vdash P \approx Q :: x{:}A$ if, once composed with requirements Γ and Δ, they perform the same actions on x (as described by A).

A Behavioral Characterization of Swapping

Theorem

If $G_1 \simeq_{\mathtt{sw}} G_2$ *then* $\Gamma; \Delta \vdash \mathsf{M}[\![G_1]\!] \approx \mathsf{M}[\![G_2]\!] :: -{:}\mathbf{1}$.

- A semantic justification of key structural identities on global types
- Useful to relax sequential constraints induced by process structure
- The converse does not hold in general. Example:

$$G = \mathtt{p} \twoheadrightarrow \mathtt{q}{:}\Big\{\mathtt{l}_i\langle U_i\rangle.\mathtt{r} \twoheadrightarrow \mathtt{p}{:}\{\mathtt{l}'_j\langle U'_j\rangle.G_{ij}\}_{j\in J}\Big\}_{i\in I}$$

It cannot be swapped and yet prefixes for $\mathtt{q}$ and $\mathtt{r}$ in $\mathsf{M}[\![G]\!]$ could be commuted.

Operational Correspondence

- A formal connection between MPSTs and mediums.
  Intuition: the medium faithfully mirrors the choreography.

- The annotated medium of a global type G, denoted $\mathcal{M}[\![G]\!]_k$, uses a session on fresh name k to mimic each action of G.

- The correspondence can then be recasted as follows:
  If G is well-formed then we have the type judgment, for some Γ:

$$\Gamma; c_1{:}\langle\!\langle G\upharpoonright \mathtt{p}_1\rangle\!\rangle, \ldots, c_n{:}\langle\!\langle G\upharpoonright \mathtt{p}_n\rangle\!\rangle \vdash \mathcal{M}[\![G]\!]_k :: k{:}(\!|G|\!)$$

  $(\!|G|\!)$ denotes a binary type that captures the sequentiality in G.

- Let $S = (\boldsymbol{\nu}\widetilde{c})(P_1 \mid \cdots \mid P_n \mid \mathcal{M}[\![G]\!]_k)$ be a system realizing G.
- Every move of G can be mimicked by an action of S on k.

Outline

Concluding Remarks (1)

- Medium processes define a simple characterization of the multiparty interactions that underlie actual choreographic protocols
- They offer a formal connection between typed frameworks for multiparty and binary communications
- Not merely a pleasant reduction: our approach establishes a natural bridge between session types and well-established theories

Concluding Remarks (2)

- Logically motivated BSTs reveal strong and tight correspondences between typed mediums and the local projections of a global type.
- These correspondences are useful! Key guarantees
  - ⋆ preservation
  - ⋆ progress / lock-freedom
  - ⋆ termination
  - ⋆ behavioral equivalences

  can be transferred from BSTs to MPSTs.

Concluding Remarks (2)

- Logically motivated BSTs reveal strong and tight correspondences between typed mediums and the local projections of a global type.
- These correspondences are useful! Key guarantees
  - ⋆ preservation
  - ⋆ progress / lock-freedom
  - ⋆ termination
  - ⋆ behavioral equivalences

  can be transferred from BSTs to MPSTs.
- Moreover, logically motivated theories of BSTs with
  - ⋆ recursion [Toninho et al., 2014]
  - ⋆ asynchrony [DeYoung et al., 2012]
  - ⋆ dependent types [Toninho et al., 2011]
  - ⋆ parametric polymorphism [Caires et al., 2013]
  - ⋆ ...

  can be lifted to MPSTs!

Bridging the Gap Between Binary and Multiparty Communications

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Open Problems in Concurrency Theory (OPCT)
Bertinoro, June 2014

Session types as linear logic propositions

The type syntax coincides with dual intuitionistic linear logic:

$$
\begin{aligned}
A, B \quad ::= {} & \mathbf{1} \mid A \otimes B \mid A \multimap B \mid !A \\
\mid {} & \&\{\mathtt{l}_i : A_i\}_{i \in I} \mid \oplus\{\mathtt{l}_i : A_i\}_{i \in I}
\end{aligned}
$$

[No atomic formulas, $\top$, $\mathbf{0}$]

Session types as linear logic propositions

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[No atomic formulas, $\top$, $\mathbf{0}$]

Types are assigned to names and describe their session behavior:

$x : A \otimes B$	Output an A along x and behave as B on x
$x : A \multimap B$	Input an A along x and behave as B on x
$x : !A$	Persistently offer A along x
$x : \&\{\mathtt{l}_i : A_i\}_{i \in I}$	Offer a choice between an A_i along x
$x : \oplus\{\mathtt{l}_i : A_i\}_{i \in I}$	Select one of the A_i along x
$x : \mathbf{1}$	Terminated interaction on x

Local Types and Logic-Based Types

Definition

The mapping $\langle\!\langle \cdot \rangle\!\rangle$ from local types T into binary session types A is inductively defined as:

$$
\begin{aligned}
\langle\!\langle \mathsf{end} \rangle\!\rangle &= \mathbf{1} \\
\langle\!\langle \mathtt{p}!\{\mathtt{l}_i\langle U_i\rangle.T_i\}_{i\in I} \rangle\!\rangle &= \oplus\{\mathtt{l}_i : U_i \otimes \langle\!\langle T_i \rangle\!\rangle\}_{i\in I} \\
\langle\!\langle \mathtt{p}?\{\mathtt{l}_i\langle U_i\rangle.T_i\}_{i\in I} \rangle\!\rangle &= \&\{\mathtt{l}_i : U_i \multimap \langle\!\langle T_i \rangle\!\rangle\}_{i\in I}
\end{aligned}
$$

Annotated Mediums

Definition

Let G be a global type. Also, let k be a fresh name.
The annotated medium of G with respect to and k, denoted $\mathcal{M}[\![G]\!]_k$, is defined inductively as follows:

- $\mathcal{M}[\![\mathtt{end}]\!]_k = \mathbf{0}$
- $\mathcal{M}[\![\mathsf{p} \twoheadrightarrow \mathsf{q}{:}\{\mathtt{l}_i\langle U_i\rangle.G_i\}_{i\in I}]\!]_k =$
$$c_{\mathsf{p}} \triangleright \Big\{ \mathtt{l}_i : \; k \triangleleft \mathtt{l}_i ; c_{\mathsf{p}}(u).\overline{k}(\mathsf{p}).\big(\mathbf{0}_{\mathsf{p}} \mid c_{\mathsf{q}} \triangleleft \mathtt{l}_i ; k \triangleright \{\mathtt{l}_i : \overline{c_{\mathsf{q}}}(v).([u \leftrightarrow v] \mid k(\mathsf{q}).\mathcal{M}[\![G_i]\!]_k)\}_{\{i\}}\big)\Big\}_{i\in I}$$

where p and q are names assumed distinct from any other name c_{p_i}.

Global Types and Logic-Based Types

Definition

Let $\sigma(\cdot)$ denote a mapping from participants to logic-based types. The mapping $(\!|\cdot|\!)$ from global types G into binary session types A is inductively defined as:

$$
\begin{aligned}
(\!|\mathtt{end}|\!) &= \mathbf{1} \\
(\!|\mathtt{p} \twoheadrightarrow \mathtt{q}{:}\{\mathtt{l}_i\langle U_i\rangle.G_i\}_{i\in I}|\!) &= \oplus\{\mathtt{l}_i : \sigma(\mathtt{p}) \otimes \&\{\mathtt{l}_i : \sigma(\mathtt{q}) \multimap (\!|G_i|\!)\}_{\{i\}}\}_{i\in I}
\end{aligned}
$$

Medium of a Global Type with Recursion

Definition

Let G be a global type with recursion $\mu\mathcal{X}.G$. Also, let $\mathtt{l}$ be a label. The *medium* of G with respect to $\mathtt{l}$, noted $\mathsf{M}[\![G]\!]^{\mathtt{l}}$, is defined inductively as follows:

- $\mathsf{M}[\![\mathtt{end}]\!]^{\mathtt{l}} = k \triangleleft \mathtt{l}; \mathbf{0}$
- $\mathsf{M}[\![\mathsf{p} \twoheadrightarrow \mathsf{q}{:}\{\mathtt{l}_i\langle U_i\rangle.G_i\}_{i\in I}]\!]^{\mathtt{l}'} =$
  $$c_{\mathtt{p}} \triangleright \big\{\mathtt{l}_i : c_{\mathtt{p}}(u).c_{\mathtt{q}} \triangleleft \mathtt{l}_i; \overline{c_{\mathtt{q}}}(v).([u \leftrightarrow v] \mid \mathsf{M}[\![G_i]\!]^{\mathtt{l}_i})\big\}_{i\in I}$$
- $\mathsf{M}[\![\mu\mathcal{X}.G]\!]^{\mathtt{l}} = (\mathbf{corec}\,\mathcal{X}(k).\mathsf{M}[\![G]\!]^{\mathtt{l}})\,k$
- $\mathsf{M}[\![\mathcal{X}]\!]^{\mathtt{l}} = k \triangleleft \mathtt{l}; \mathcal{X}(k)$

where name k is assumed to be distinct from any other name $c_{\mathtt{p}_i}$.