Synthesis of Concurrent Systems with Step Firing Policies

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Bertinoro, June 2014

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Synthesis technique: our focus

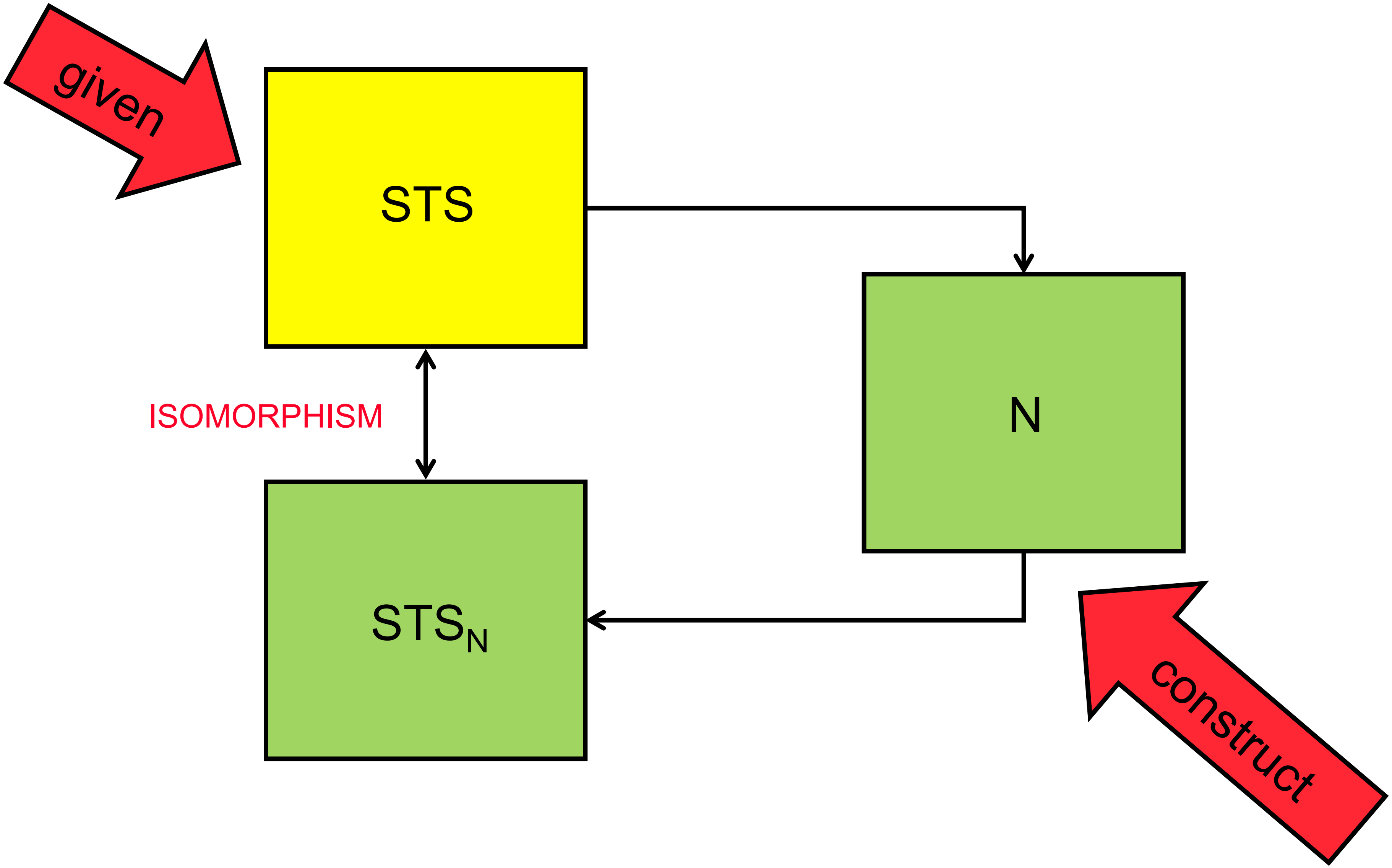
- Semantics based on steps: groups of transitions executed simultaneously
- Construction of Petri nets from step transition systems
- Steps enabled by the structure of a net are further constrained through ***step firing policies*** reflecting:

  maximal parallelism, priorities, required energy, etc

- Foundations of such synthesis developed in

  [Ph.Darondeau, M.K, M.P-K, A.Yakovlev, 2008/09]

Synthesis problem



PTL-nets: nets with firing policy

PTL–net (PT-net with localities)

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PT-net (P,T,W,M_0)

+

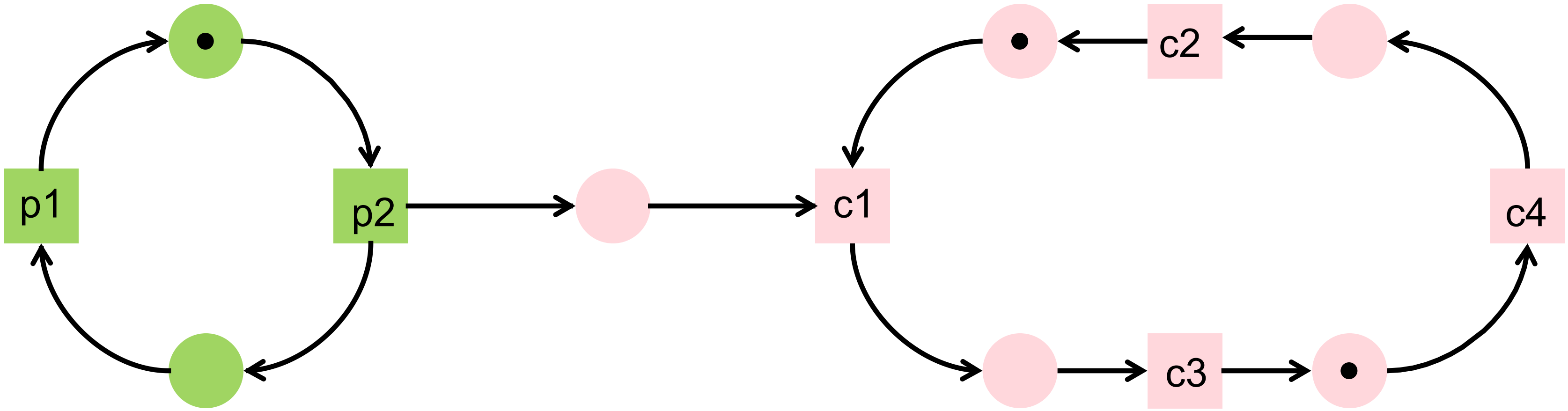
co-location relation ≏ for transitions and places

- If x ≏ z then x and z are co-located
- Transitions residing at a locality are executed in maximally concurrent manner

Locally maximal firing policy: lmax

Step sequences for one producer and two co-located consumers

- {p2}{c1} is *illegal*: lmax violated by {c1}
- {p2}{c1, c4} & {p2}{c1, c4, p1} are *legal*



Applications of PTL-nets

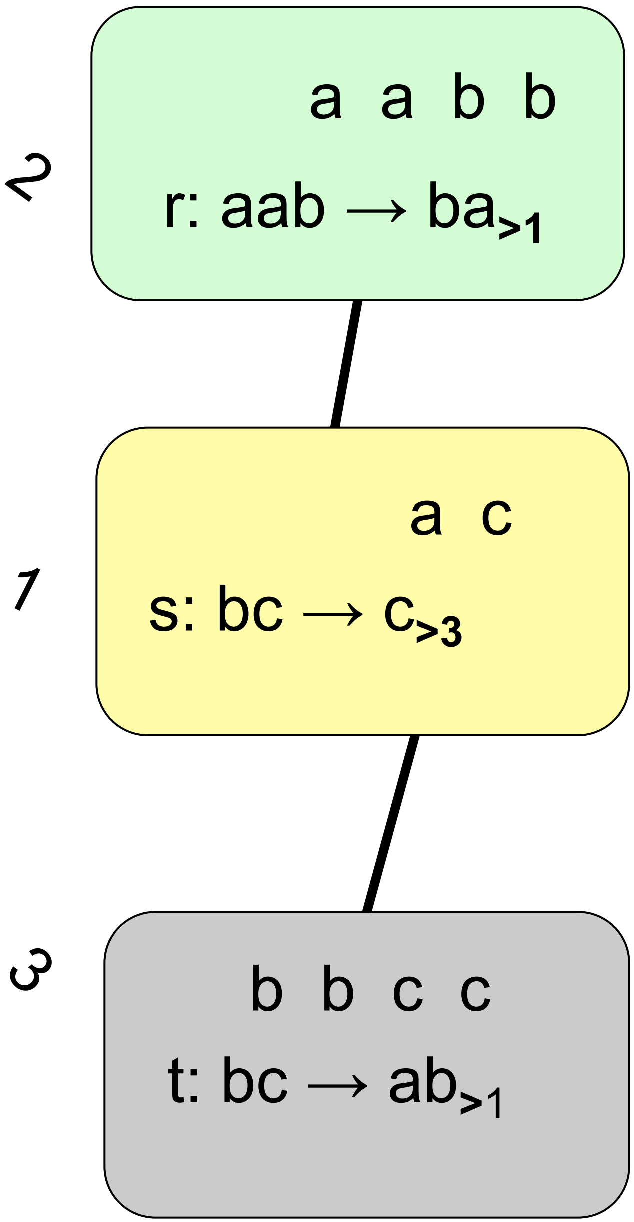
- PTL-nets can model globally asynchronous locally synchronous systems (GALS)

- Examples:
  - VLSI chips with multiple clocks for synchronisation of different subsets of gates
  - Membrane systems modelling cells inside which reactions are carried out in co-ordinated pulses
  - Tissue systems

Tissue systems

- Formal computational model inspired by compartments and functionality of living cells
- Biochemical reactions take place in compartments
- Compartments are determined by the structure of membranes (can be porous)
- Biochemical reactions represented by rewriting rules

Tissue systems



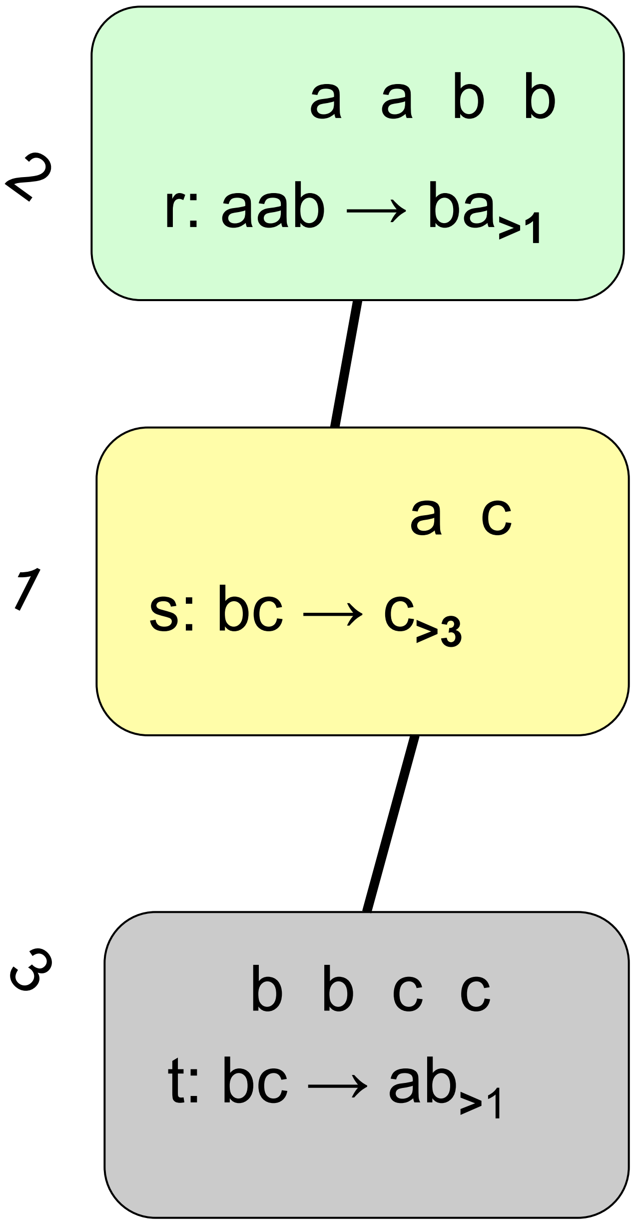
alphabet
tissue structure
initial objects
evolution rules
effect of individual occurrence

fixed structure

no exchange of objects with the external environment

...

Executed steps: multisets of rules

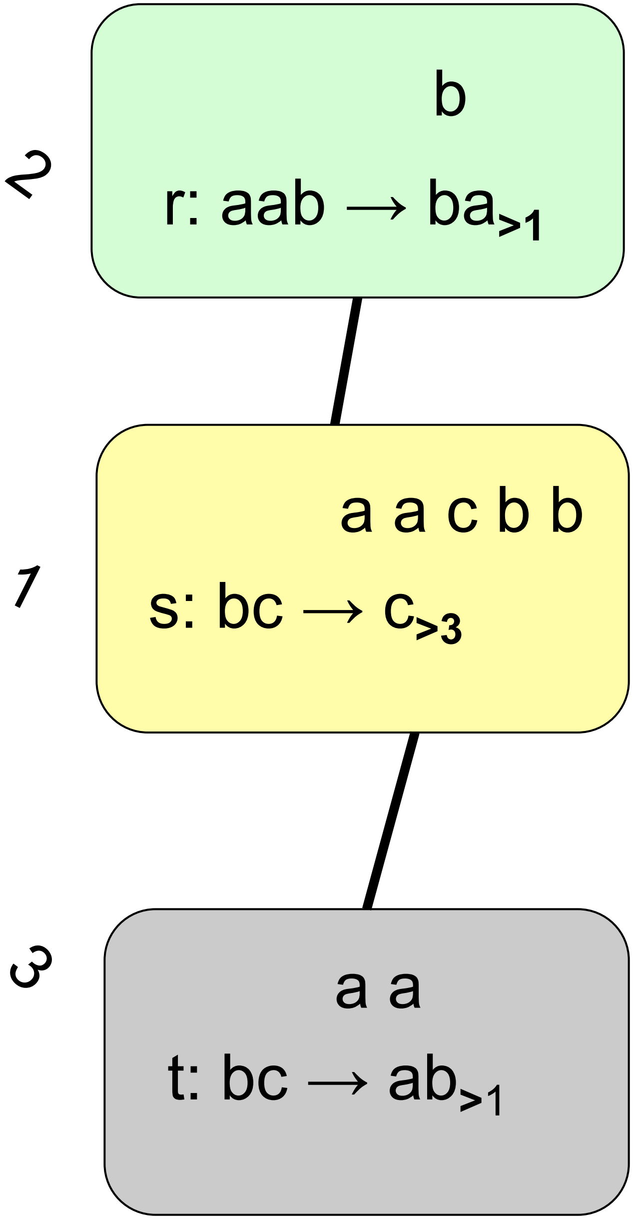


Finite and infinite sequences of executed steps

{r,t} is illegal

{r,t,t} is legal and leads to

Executed steps: multisets of rules



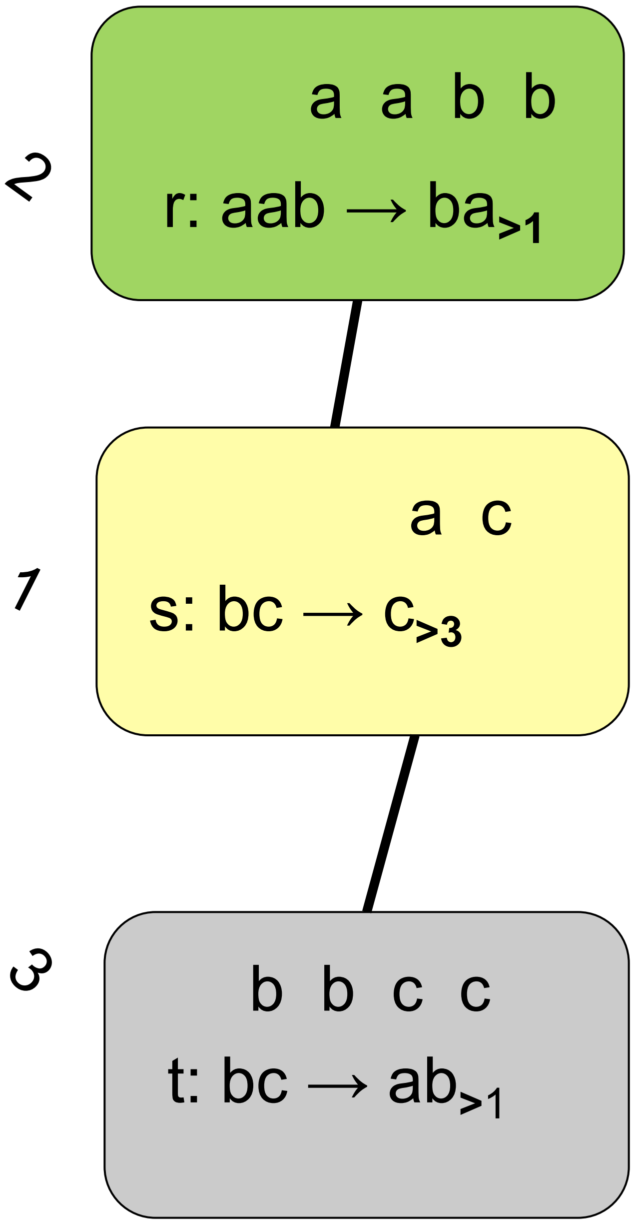
Finite and infinite sequences of executed steps

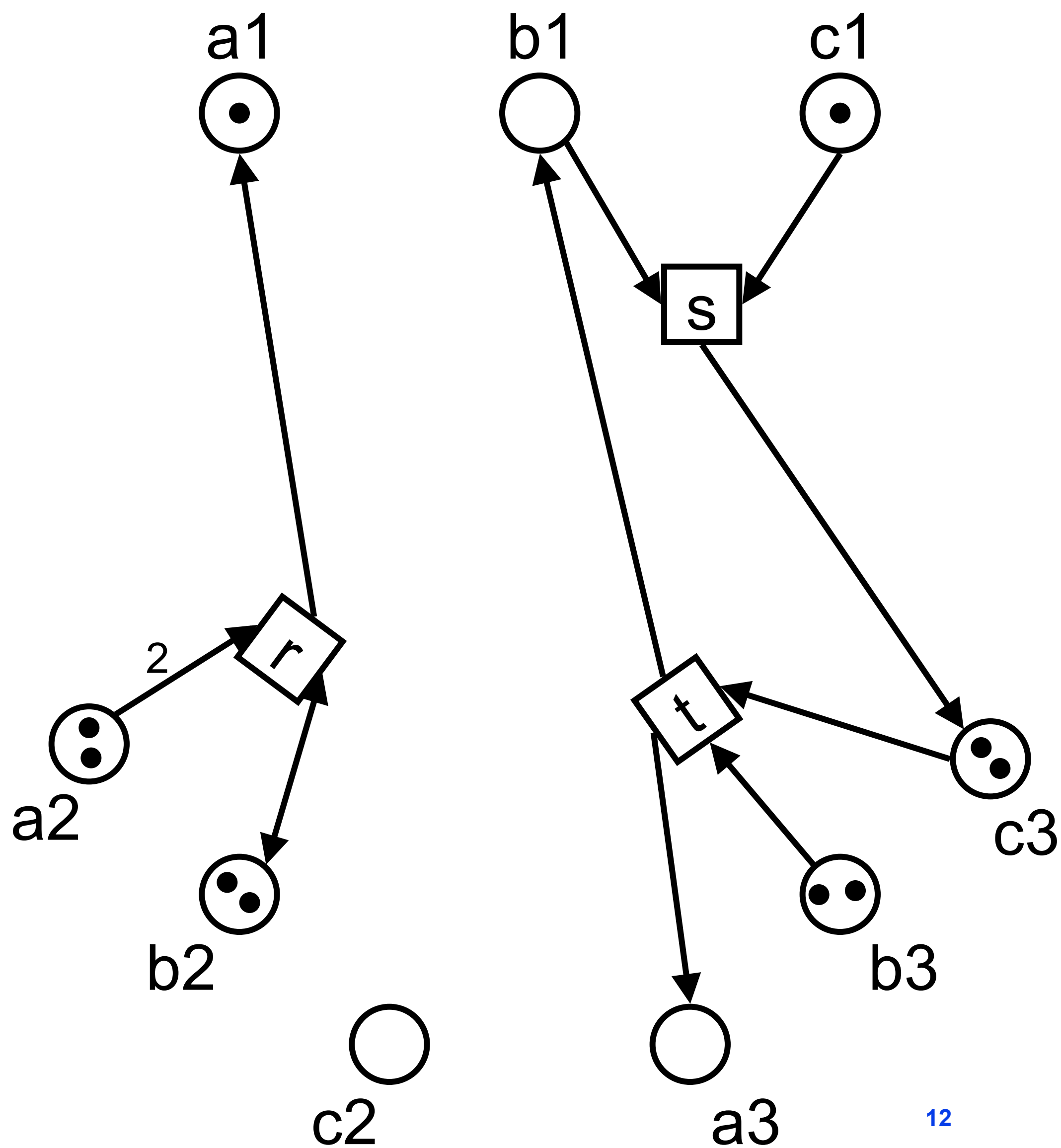
{r,t} is illegal

{r,t,t} is legal and leads to

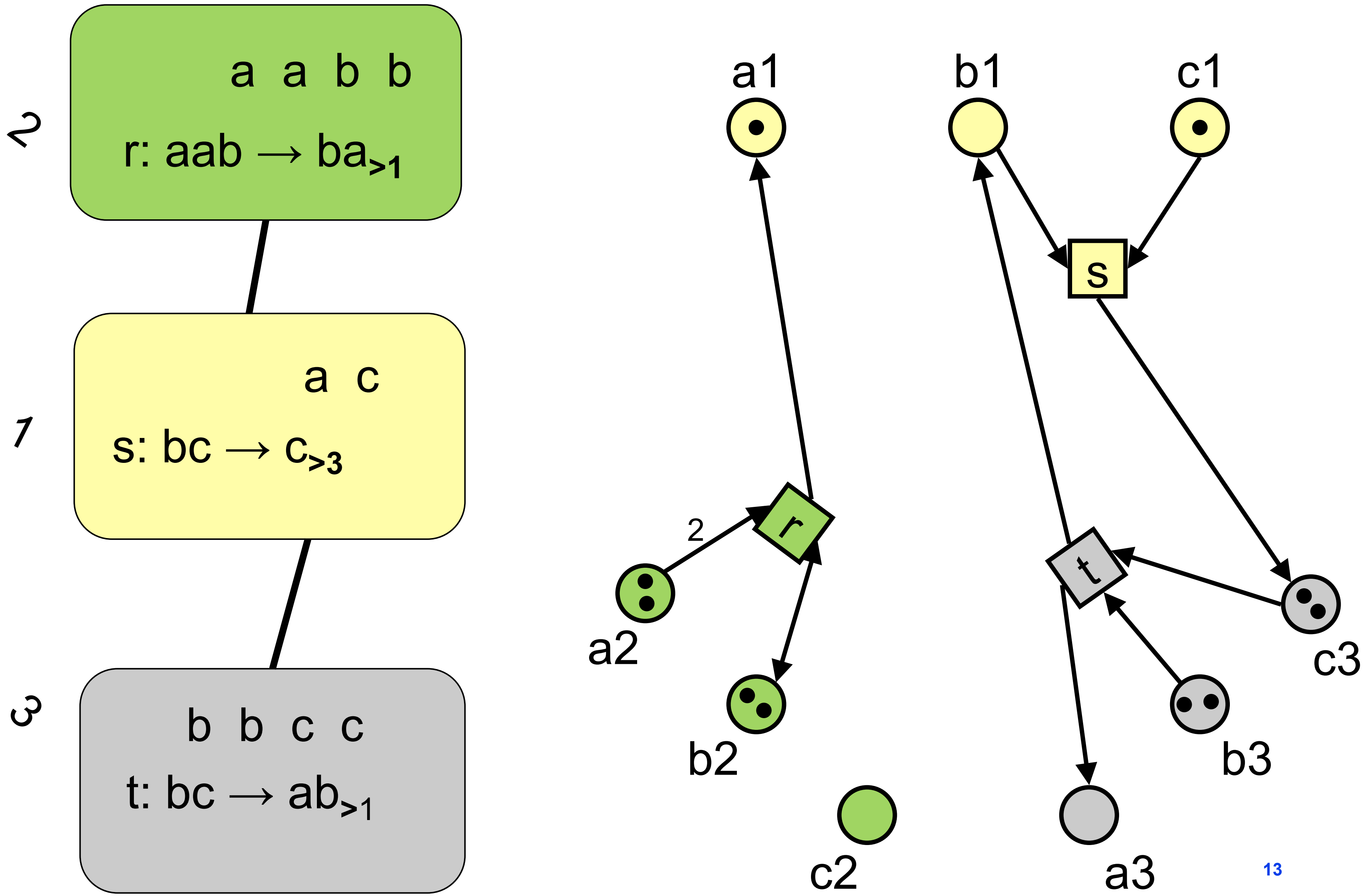
Tissue systems and PTL-nets

Tissue system as PTL-net





Tissue system as PTL-net



Tissue system as PTL-net

PTL-net is spanned over the tissue structure:

- input only from the same locality
- output to the same locality or the neighbours

local resource	corresponds to	token in place
rule	corresponds to	transition

Imax executions of tissue system and the corresponding PTL-net generate isomorphic step transition systems

synthesis of tissue systems

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synthesis of PTL-nets spanned over tissue structures

Step transition system

- Behavioural model for PTL-nets
  - states (Q)
  - transitions/arcs (A)
  - initial state (q_0)

- Arcs labelled by multi-sets of (net) transitions executed simultaneously

Synthesis problem A

INPUT

finite step transition system STS with transitions T

tissue structure τ

co-location relation $\hat{=}$ for T

OUTPUT (if possible)

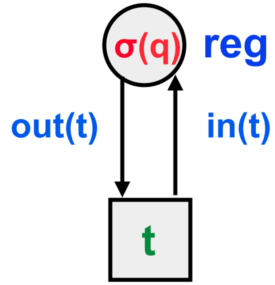
finite PTL-net N:

- N spanned over τ and respecting $\hat{=}$
- STS isomorphic to CRG(N)

Synthesis problem A

A region is

$$reg=(\sigma: Q \rightarrow \mathbb{N},\ in: T \rightarrow \mathbb{N},\ out: T \rightarrow \mathbb{N})$$



such that, for every arc (q,α,q’) of STS

$$\sigma(q) \geq out(\alpha) \text{ and } \sigma(q') = \sigma(q) - out(\alpha) + in(\alpha)$$

reg with locality i_{reg} is compatible with tissue structure τ if:

- out(t)>0 implies that t and reg are co-located
- in(t)>0 implies that t and reg are co-located or located in neighbour localities

Only compatible regions can be places in synthesised net!

Synthesis problem A

- Regions are used to check the feasibility of the synthesis problem and to construct PTL-net

- Regions can be computed [Chernikova 1965] as integer solutions $p = x_0 \ldots x_m y_1 \ldots y_n z_1 \ldots z_n$ of the following system:

  $x_i \geq \alpha.z$
  $x_j = x_i + \alpha.(y-z)$ for all arcs (q_i,α,q_j) in STS

  where $\sigma(q_i) = x_i$ and $in(t_j) = y_j$ and $out(t_j) = z_j$

- Each compatible solution p of the system above can be expressed as a non-negative linear combination of some k basic compatible integer solutions (rays):

$$p = r_1.p^1 + \ldots + r_k.p^k$$

Synthesis problem A

For STS to be synthesisable it needs to satisfy:

- State separation checked for states q_i and q_j as follows:

  Is there a ray $x_0^b \ldots x_m^b y_1^b \ldots y_n^b z_1^b \ldots z_n^b$ such that $x_i^b \neq x_j^b$?

- Forward closure (event/state separation) checked for every state q_i as follows:

  Is there a ray such that $x_i^b - \alpha . z^b < 0$?
  Yes means: α is not region enabled at q_i

  We check whether the steps on outgoing arcs are exactly region enabled steps α for which

  there is no transition t (co-located with some transition from α) such that $\alpha + t$ is region enabled at q_i

What if the tissue structure is not known?

Synthesis problem B

INPUT

finite step transition system STS with transitions T

OUTPUT (if possible)

finite PTL-net N and tissue structure τ:

- N spanned over τ
- STS isomorphic to CRG(N)

What if the tissue structure is not known?

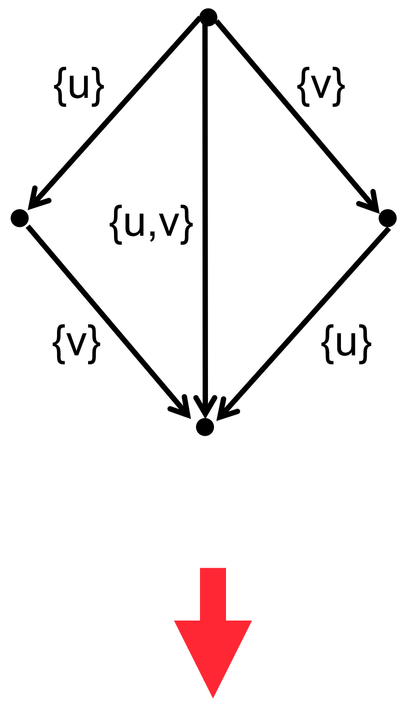
The number of co-location relations for n transitions is finite – Perhaps we can try them all ?   very expensive!

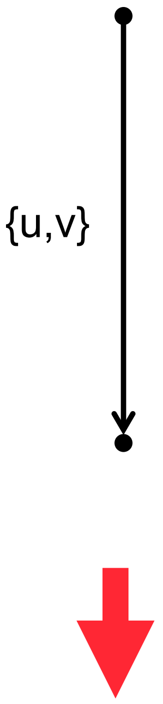
Perhaps we can deduce potential tissue structures and reduce the number of cases?

In any case we can assume that the tissue structure is a clique

Can we deduce ≐ from STS in general?

YES



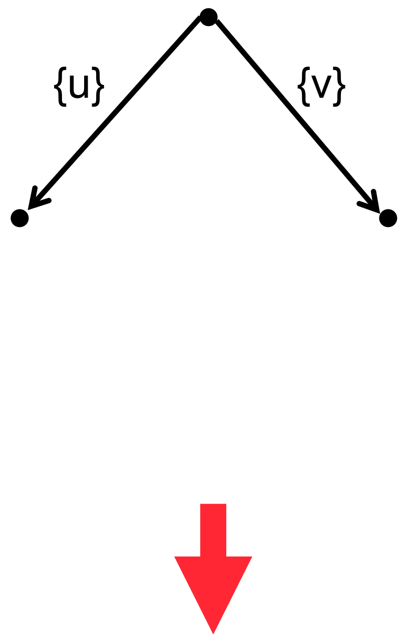


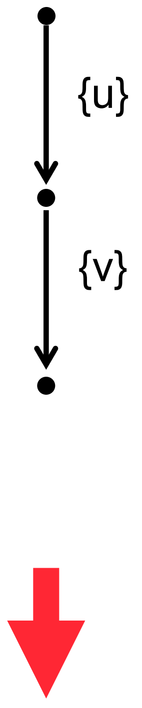
u ≠̇ v

u ≐ v

Can we deduce ≙ from STS in general?

NO



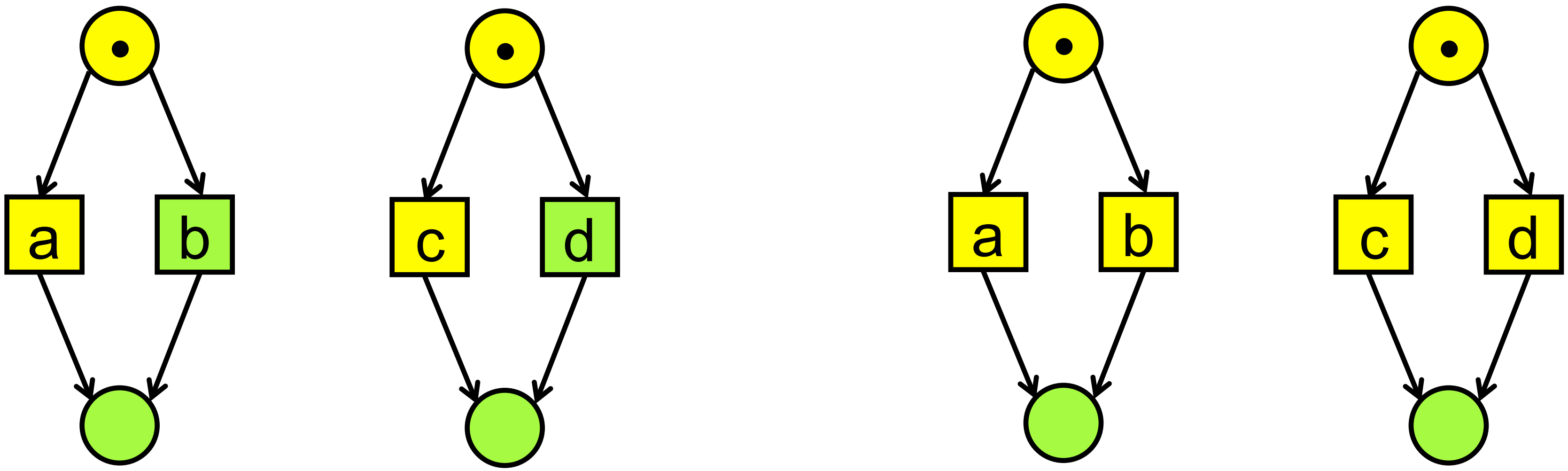


u ? v

u ? v

Can we deduce ≏ from STS in general?

PTL-nets generating the same STS



It is hard to determine co-location in the presence of conflicts

But conflicts in PTL-nets spanned over tissue structures are restricted !

PTL-nets spanned over tissue structures

A key result for PTL-nets spanned over tissue structures

two transitions enabled at marking M are co-located

if and only if

either there is no step lmax-enabled at M containing them

or there is a minimal step lmax-enabled at M containing them

Hence there is a unique co-location $\triangleq_q$ for transitions in steps labelling arcs outgoing from node q !

Synthesis Problem B

- Compute $\hat{=}_q$ for all states q of STS
- Form the transitive closure $\hat{=}_{ok}$ of their union
- Check whether each $\hat{=}_q$ is equal to projected $\hat{=}_{ok}$
  - No: Synthesis Problem B fails
  - Yes: run Synthesis Problem A for all $\hat{=}$ containing $\hat{=}_{ok}$
- If the state separation and forward closure are satisfied for any such $\hat{=}$ then the construction succeeds
- *We can further restrict ourselves to the largest (in terms of set inclusion) relations* $\hat{=}$
- *Finding the largest* $\hat{=}$ *relations is related to the minimum clique cover problem*

OPEN PROBLEMS

- Variations of the synthesis problems for PTL-nets:

  scenarios / process mining / languages …

- New approaches to handle systems exhibiting:

  locality / visibility / dynamic changes / fuzziness …

- Algorithmic efficiency through:

  modularity / hierarchy / abstraction / …

OPEN PROBLEM

Synthesis of step firing policies

Thank you!